Lecture 11

2023/2024

Microwave Devices and Circuits for Radiocommunications

2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 16-18, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

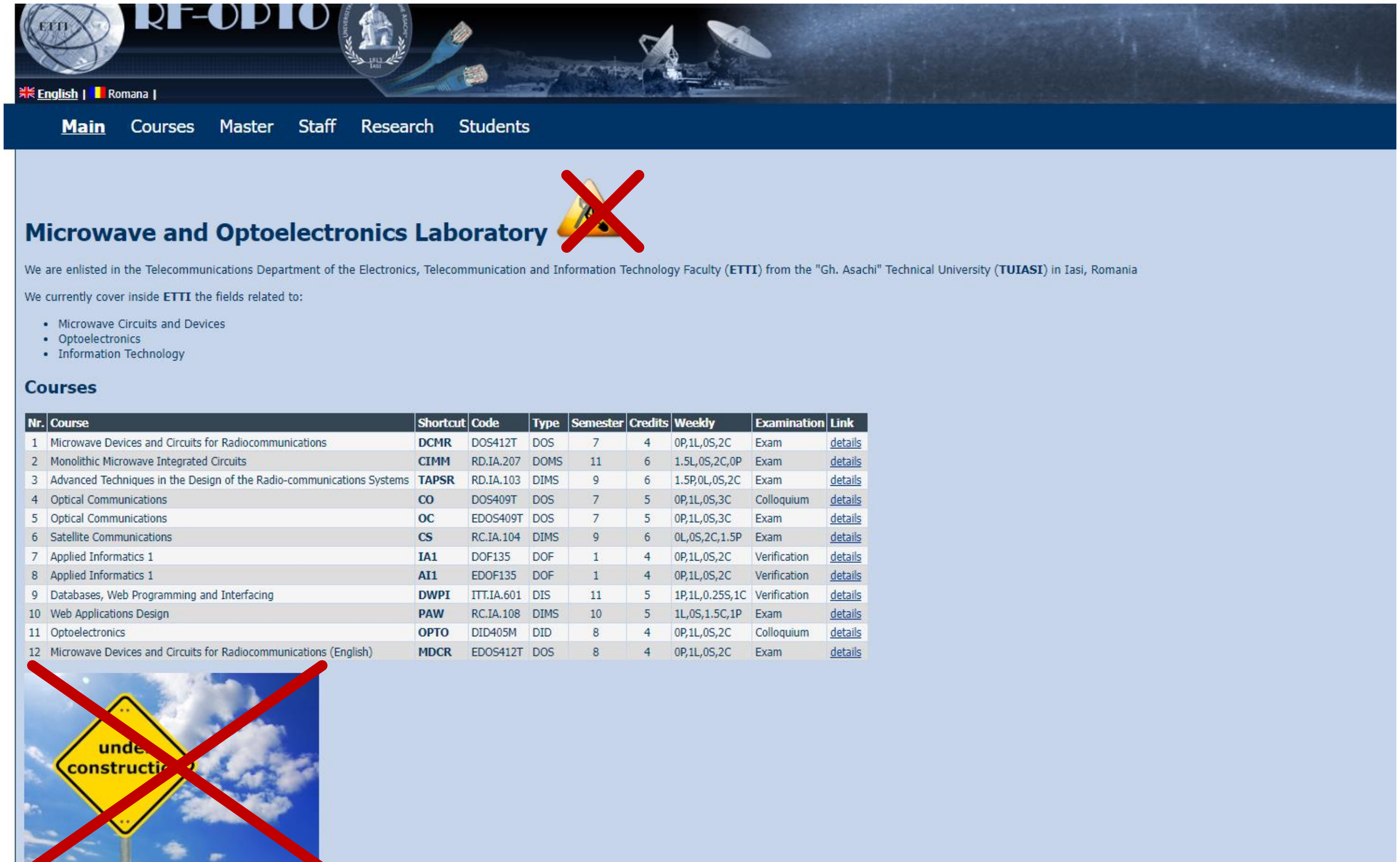
2023/2024

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 20.02.2024)
    - personal homework

Materials

- http://rf-opto.etti.tuiasi.ro

Site



English | Romana |

Main Courses Master Staff Research Students

Microwave and Optoelectronics Laboratory

We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (**ETTI**) from the "Gh. Asachi" Technical University (**TUIASI**) in Iasi, Romania

We currently cover inside **ETTI** the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

Courses

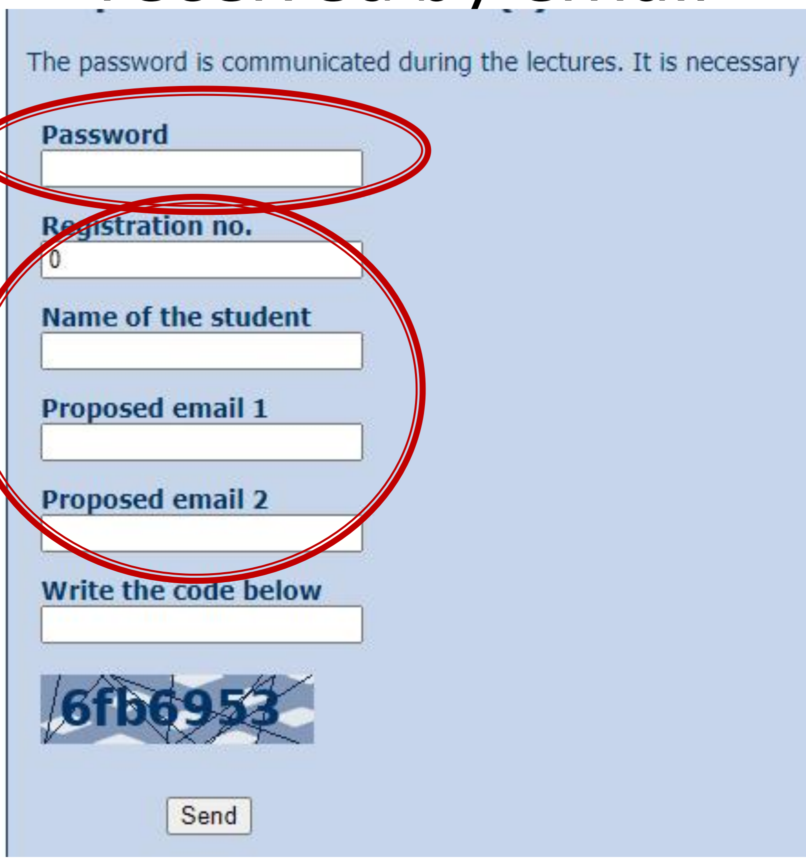
Nr.	Course	Shortcut	Code	Type	Semester	Credits	Weekly	Examination	Link
1	Microwave Devices and Circuits for Radiocommunications	**DCMR**	DOS412T	DOS	7	4	0P,1L,0S,2C	Exam	details
2	Monolithic Microwave Integrated Circuits	**CIMM**	RD.IA.207	DOMS	11	6	1.5L,0S,2C,0P	Exam	details
3	Advanced Techniques in the Design of the Radio-communications Systems	**TAPSR**	RD.IA.103	DIMS	9	6	1.5P,0L,0S,2C	Exam	details
4	Optical Communications	**CO**	DOS409T	DOS	7	5	0P,1L,0S,3C	Colloquium	details
5	Optical Communications	**OC**	EDOS409T	DOS	7	5	0P,1L,0S,3C	Exam	details
6	Satellite Communications	**CS**	RC.IA.104	DIMS	9	6	0L,0S,2C,1.5P	Exam	details
7	Applied Informatics 1	**IA1**	DOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
8	Applied Informatics 1	**AI1**	EDOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
9	Databases, Web Programming and Interfacing	**DWPI**	ITT.IA.601	DIS	11	5	1P,1L,0.25S,1C	Verification	details
10	Web Applications Design	**PAW**	RC.IA.108	DIMS	10	5	1L,0S,1.5C,1P	Exam	details
11	Optoelectronics	**OPTO**	DID405M	DID	8	4	0P,1L,0S,2C	Colloquium	details
12	Microwave Devices and Circuits for Radiocommunications (English)	**MDCR**	EDOS412T	DOS	8	4	0P,1L,0S,2C	Exam	details

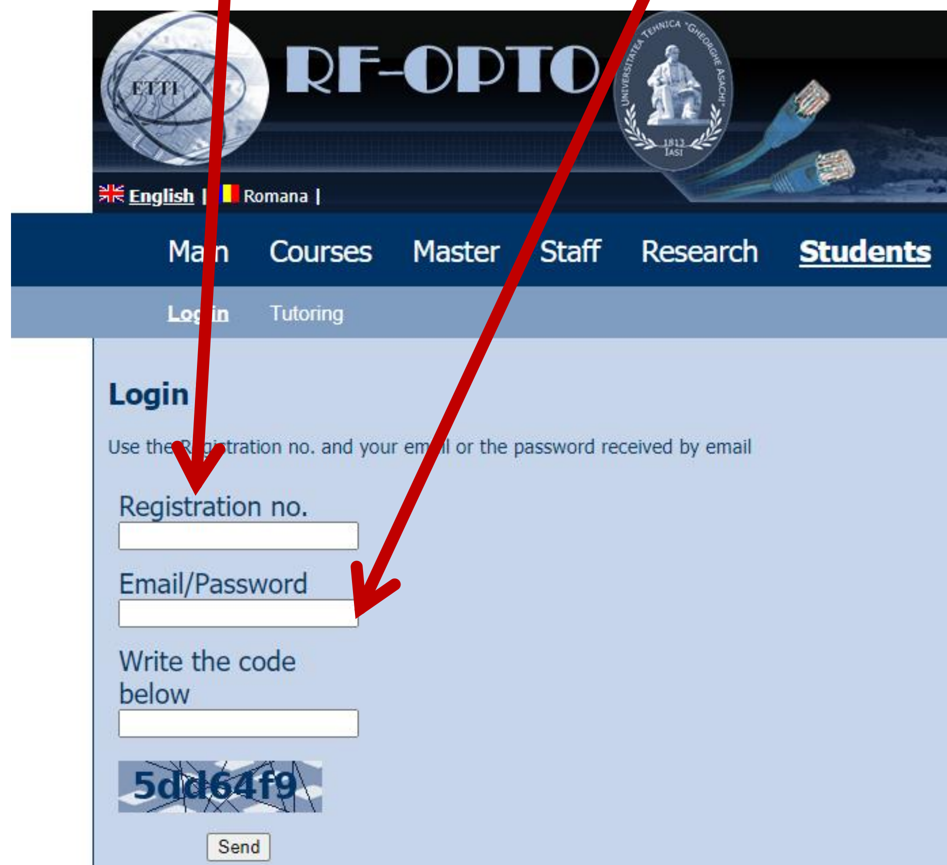
Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/**online exam > Week4-Week6**
  - used at lectures/laboratory

Online – Registration no.

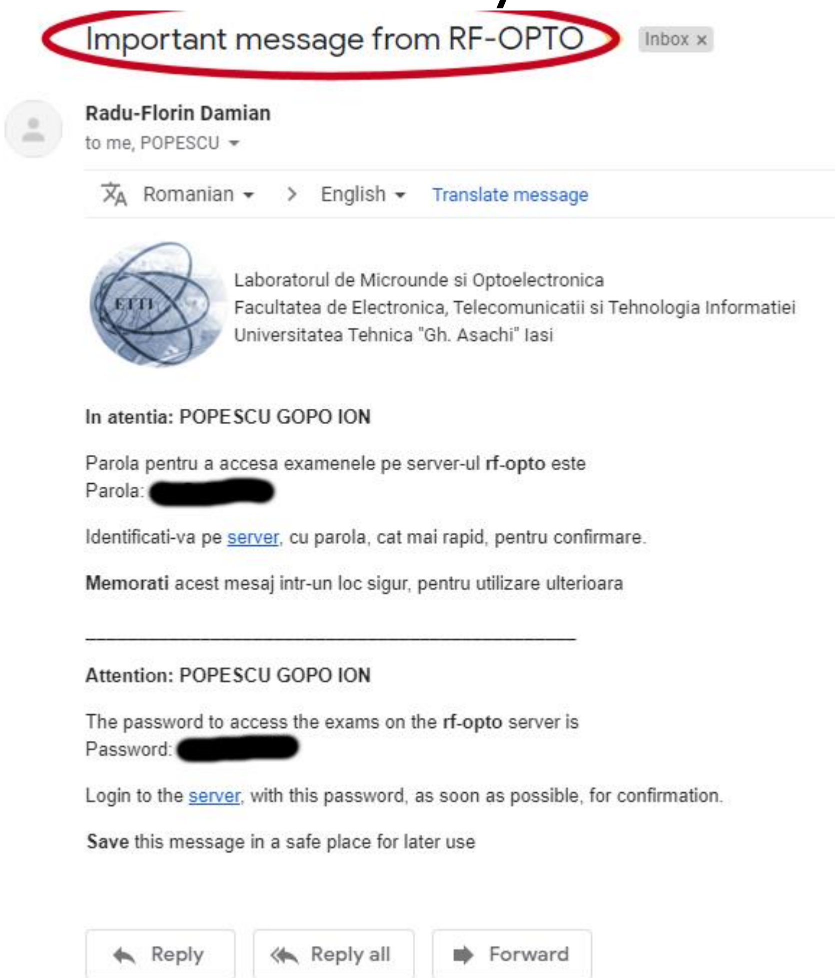
- access to **online exams** requires the **password** received by email





Password

- received by email



Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

____

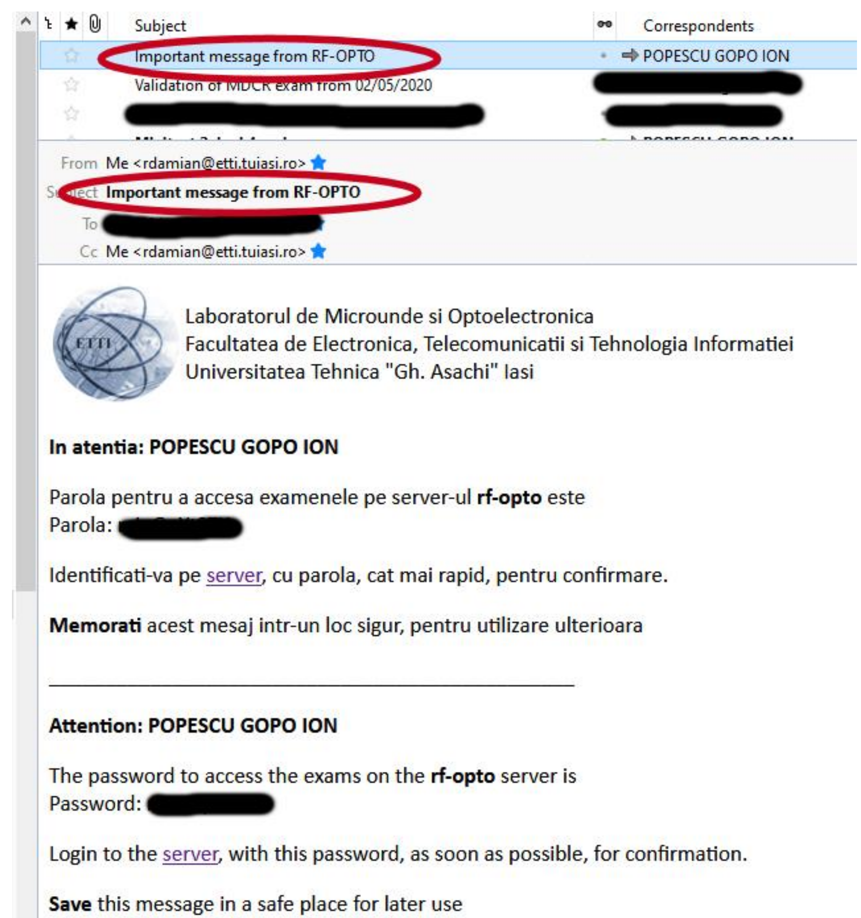
Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply | Reply all | Forward



Subject | Correspondents
Important message from RF-OPTO | POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020 | [redacted]
[redacted] | [redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

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Password: [redacted]

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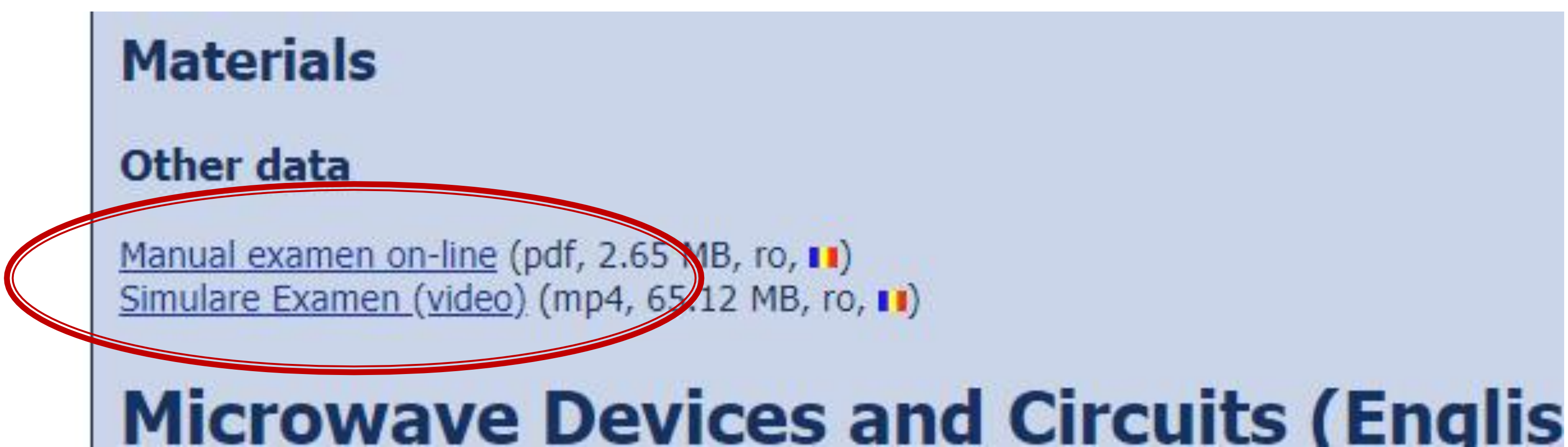
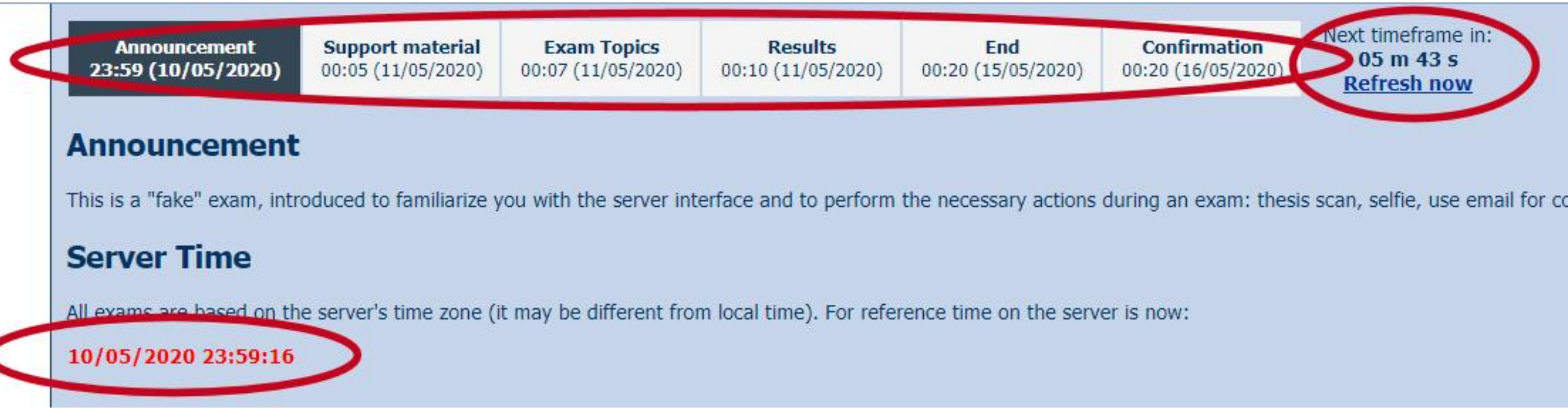
Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~



Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parametri s	T2, fisier parametri s	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

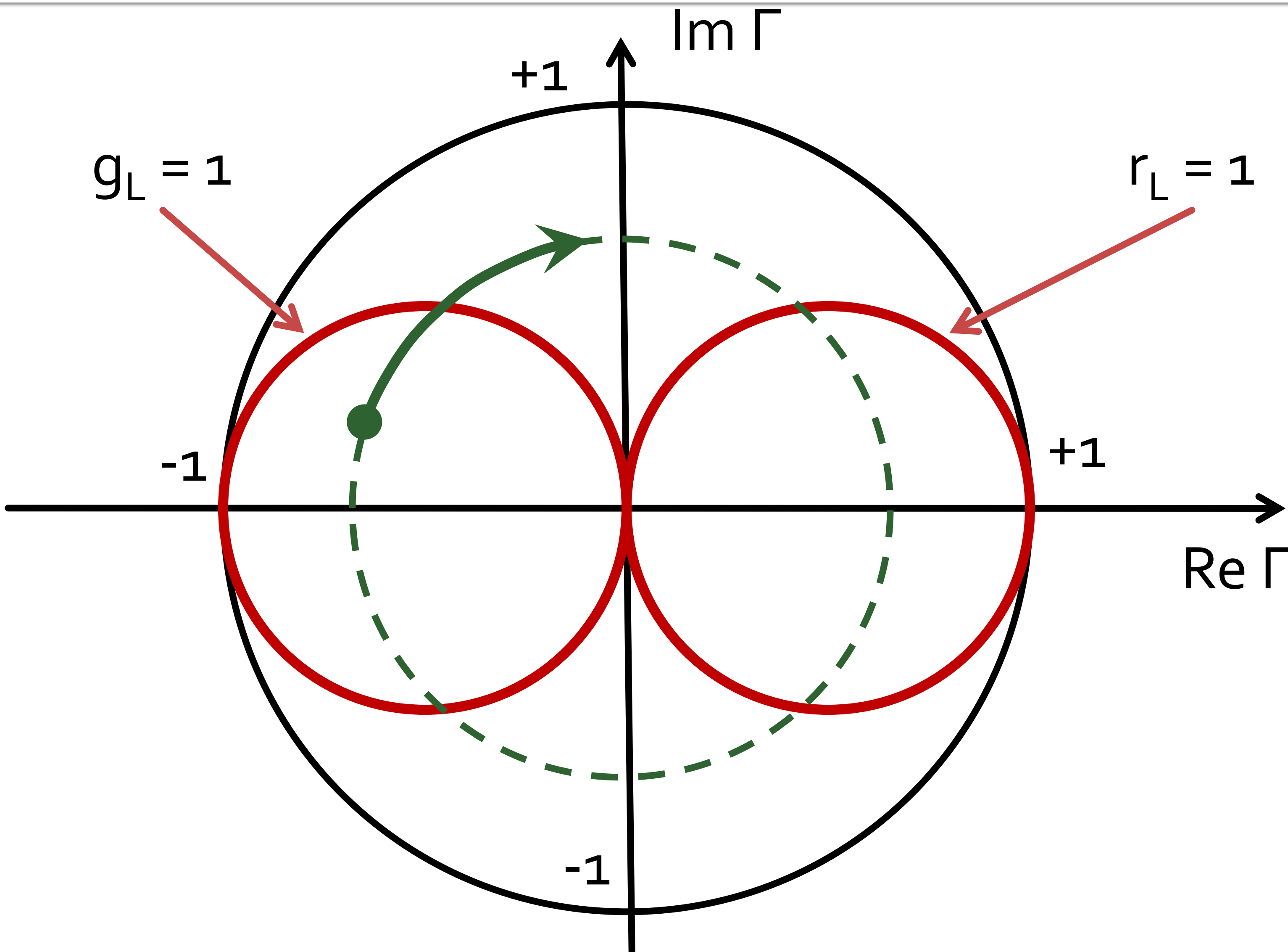
Grade = Quality of the work +

+ Quality of the submission

Impedance Matching

Impedance Matching with Stubs

Smith chart, r=1 and g=1

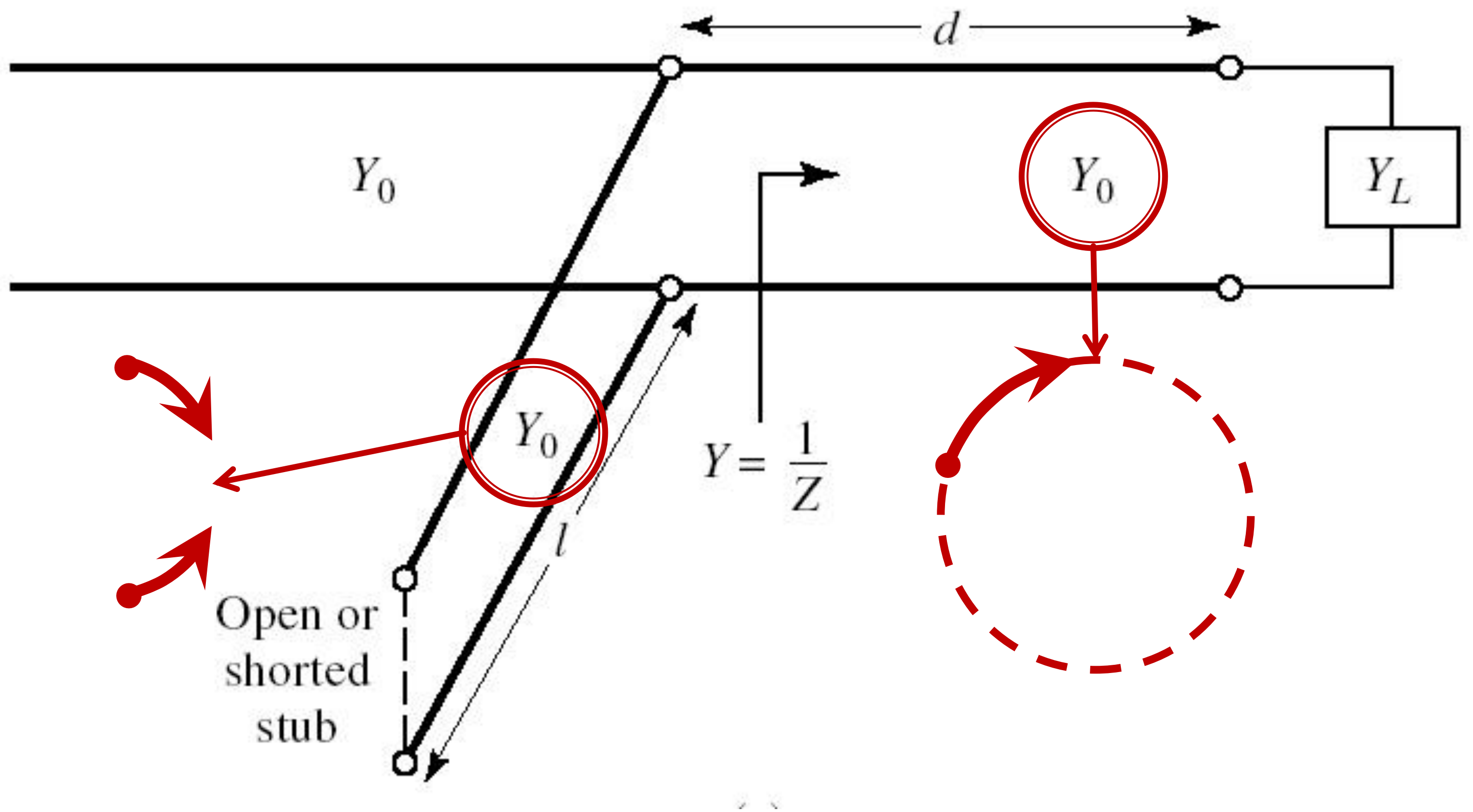


Analytical solutions

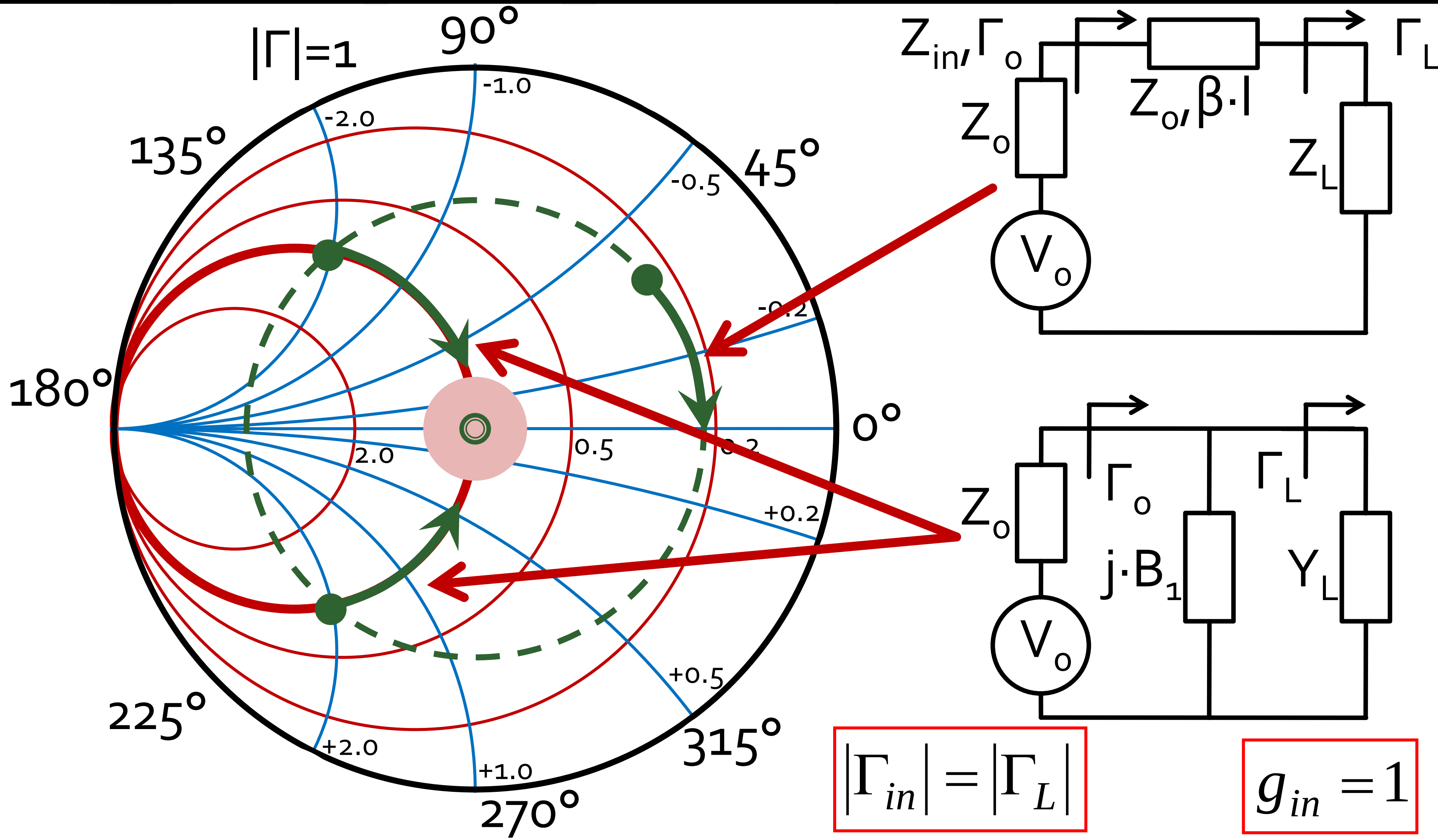
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$
  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+126.35^\circ\\-126.35^\circ\end{cases}\quad \theta=\begin{cases}39.7^\circ\\93.4^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.472\\+1.472\end{cases}\quad \theta_{sp}=\begin{cases}-55.8^\circ+180^\circ=124.2^\circ\\+55.8^\circ\end{cases}$$

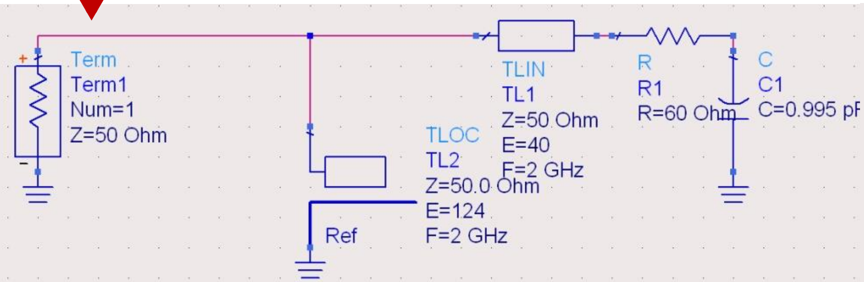
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

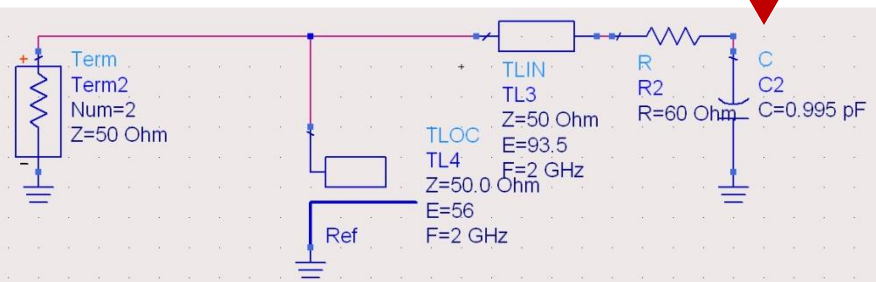
$$l_1=\frac{39.7^\circ}{360^\circ}\cdot\lambda=0.110\cdot\lambda$$

$$l_2=\frac{124.2^\circ}{360^\circ}\cdot\lambda=0.345\cdot\lambda$$

$$l_1=\frac{93.4^\circ}{360^\circ}\cdot\lambda=0.259\cdot\lambda$$

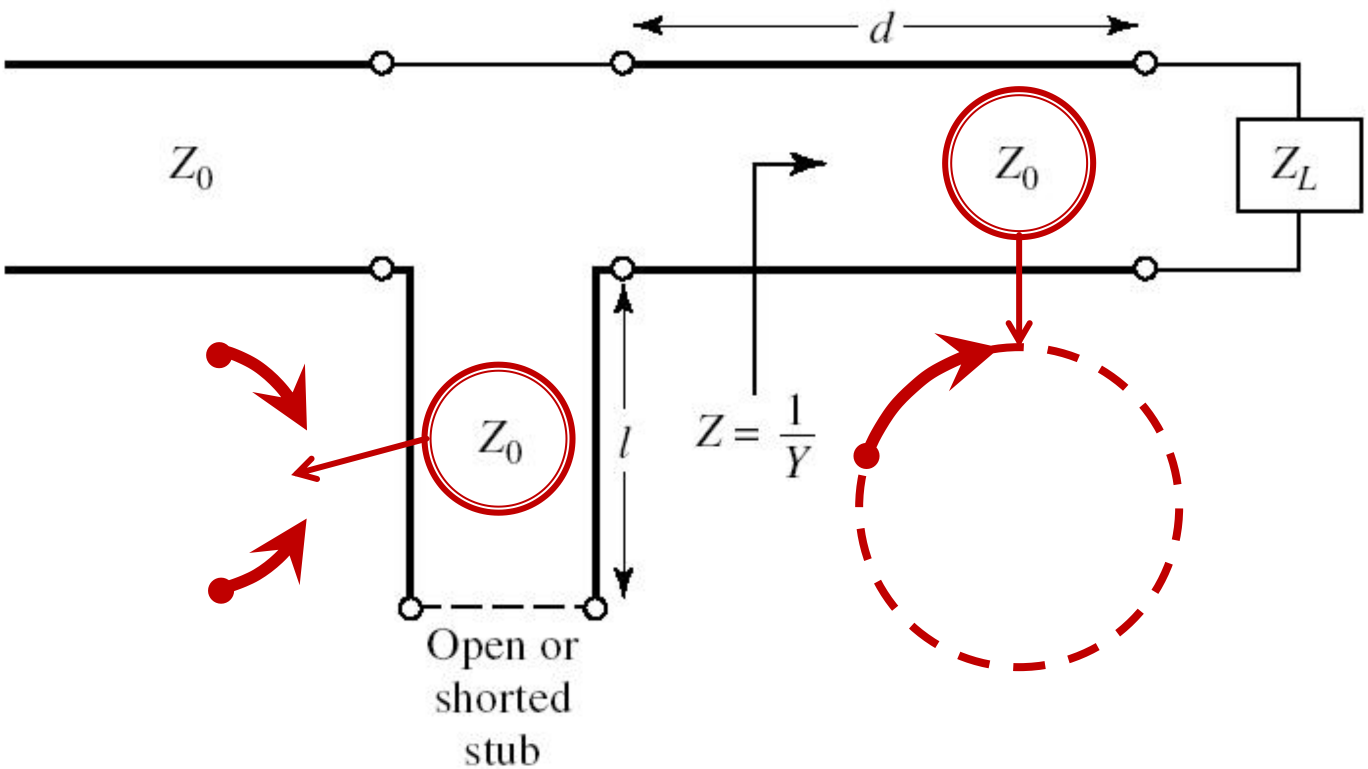
$$l_2=\frac{55.8^\circ}{360^\circ}\cdot\lambda=0.155\cdot\lambda$$



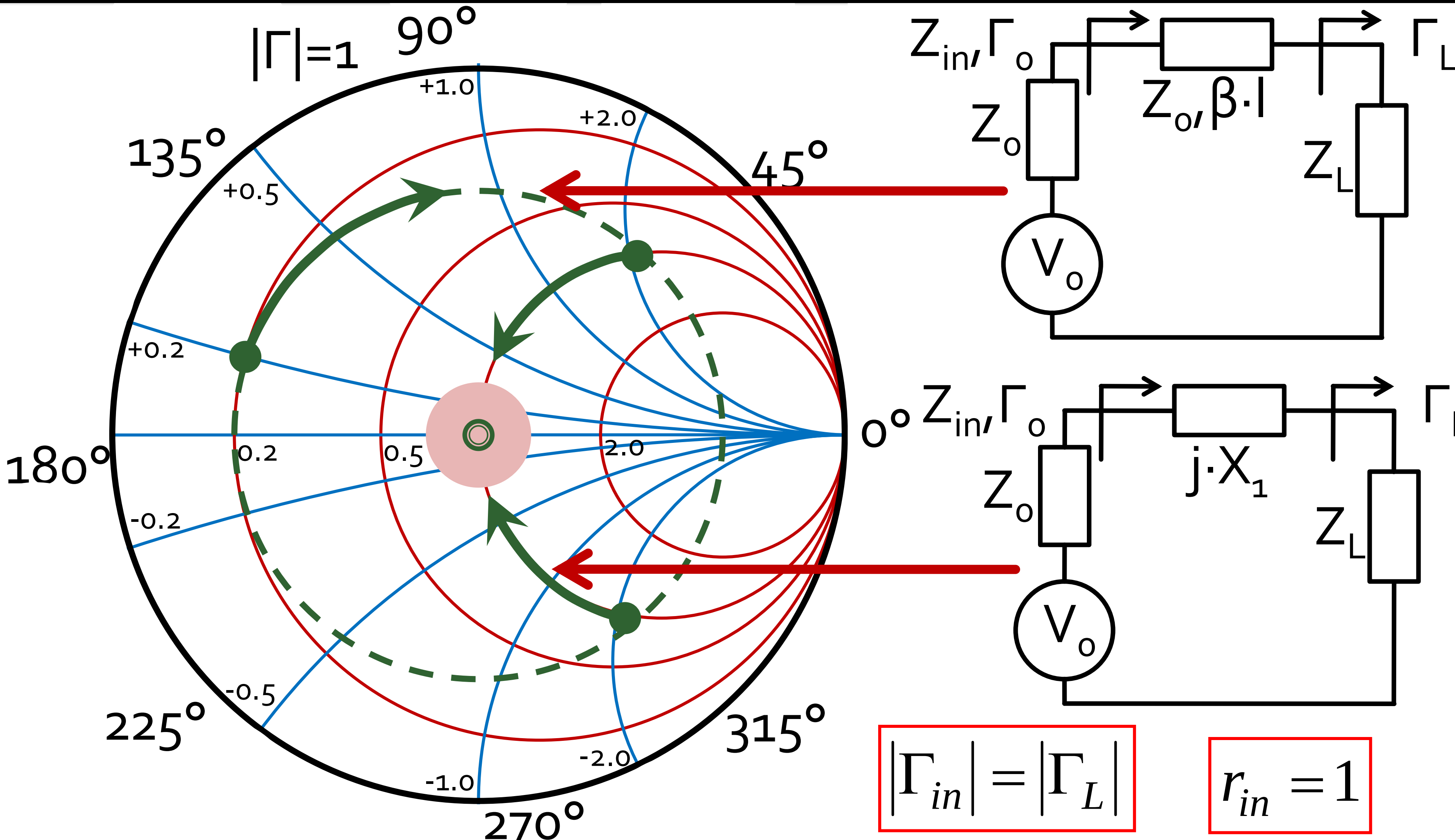


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$|\Gamma_{in}| = |\Gamma_L|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8° (+180°) \rightarrow \theta_{ss} = 143.2°$$

  - **"-" solution**

$$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2° (+180°) \rightarrow \theta = 166.8°$$

$$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8°$$

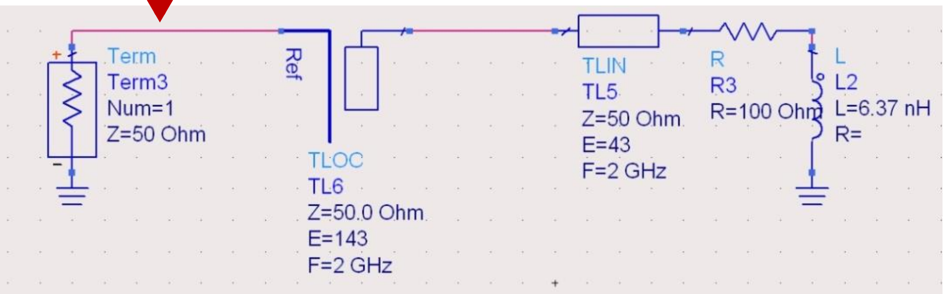
Analytical solution, usage

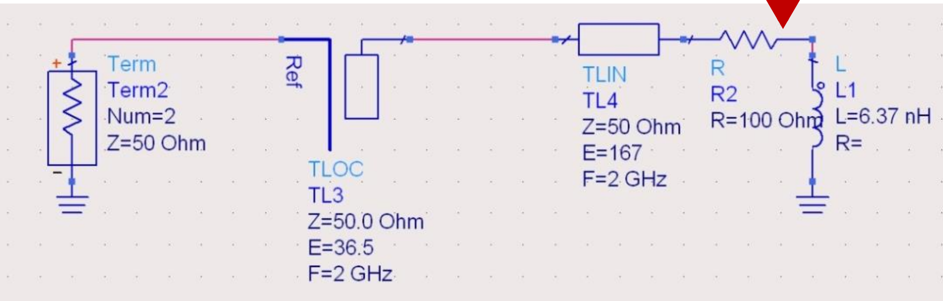
$$(\varphi+2\theta)=\begin{cases}+56.28^\circ\\-56.28^\circ\end{cases}\quad \theta=\begin{cases}43.1^\circ\\166.8^\circ\end{cases}\quad \mathrm{Im}[z_S(\theta)]=\begin{cases}+1.335\\-1.335\end{cases}\quad \theta_{ss}=\begin{cases}-36.8^\circ+180^\circ=143.2^\circ\\+36.8^\circ\end{cases}$$

- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

$$l_1=\frac{43.1^\circ}{360^\circ}\cdot\lambda=0.120\cdot\lambda$$
$$l_2=\frac{143.2^\circ}{360^\circ}\cdot\lambda=0.398\cdot\lambda$$

$$l_1=\frac{166.8^\circ}{360^\circ}\cdot\lambda=0.463\cdot\lambda$$
$$l_2=\frac{36.8^\circ}{360^\circ}\cdot\lambda=0.102\cdot\lambda$$





Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

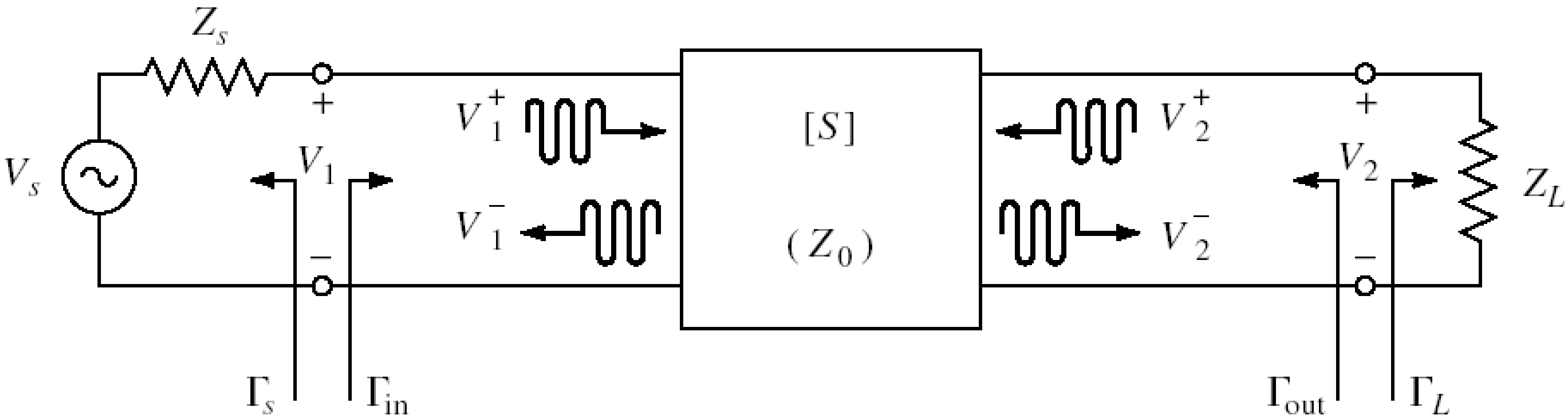
  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

Microwave Amplifiers

Amplifier as two-port

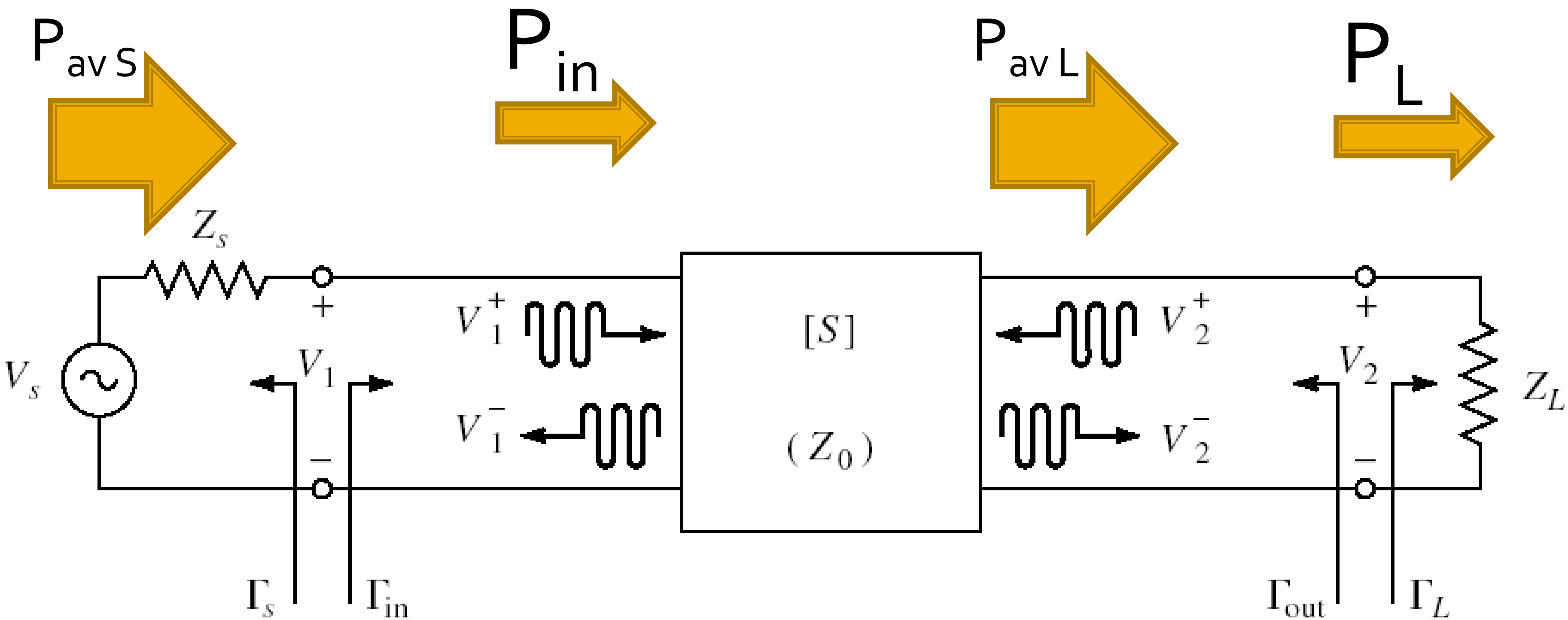


$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Power / Matching

- Two ports in which matching influences the power transfer



Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right) \cdot \left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

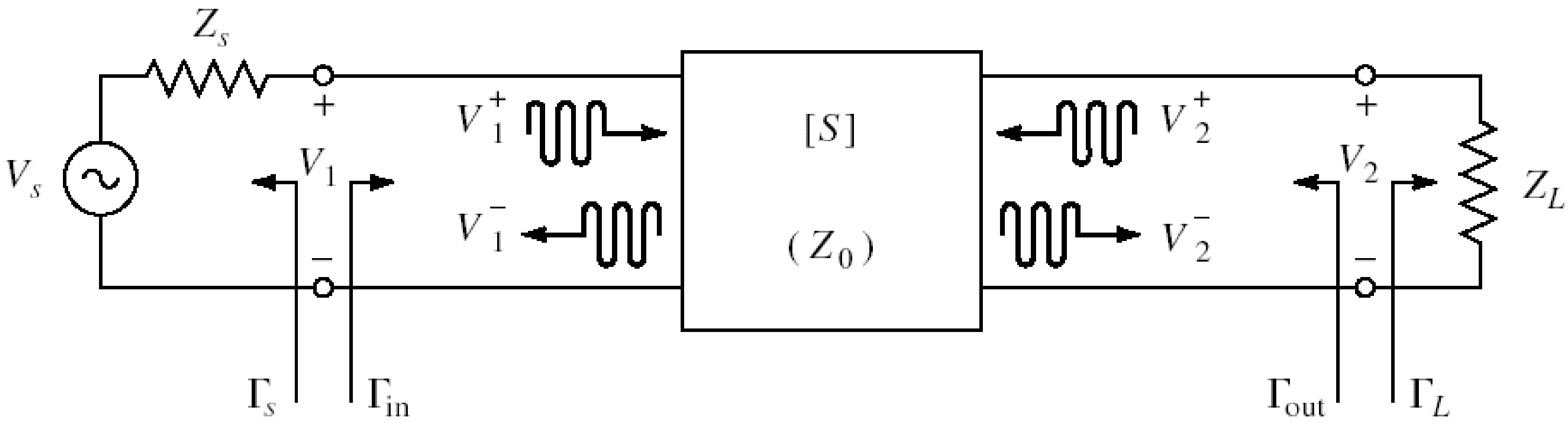
$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

Input and output can be treated independently

Microwave Amplifiers

Stability

Amplifier as two-port

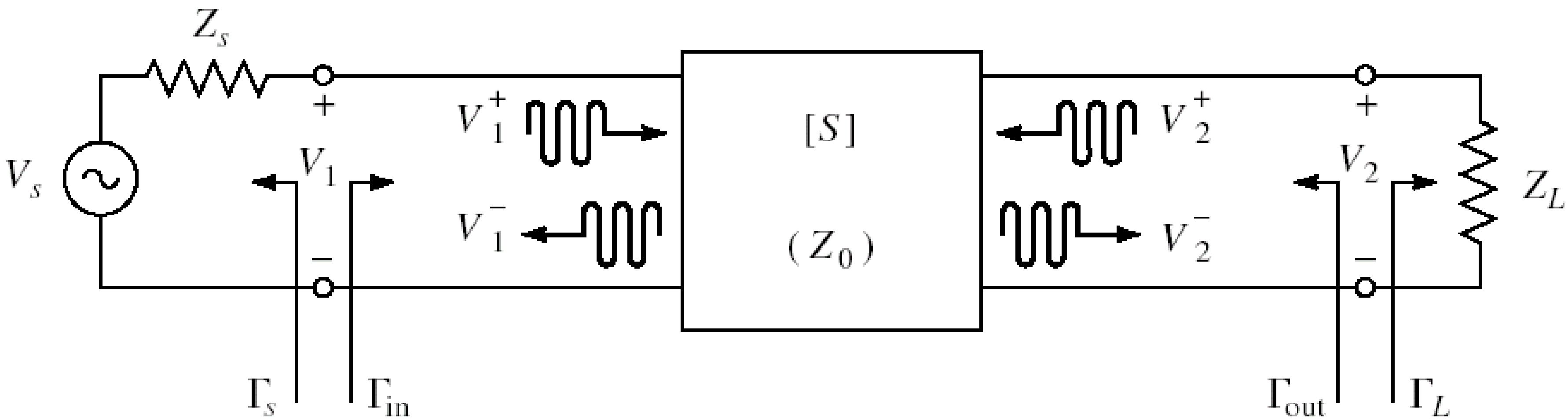


- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Microwave Amplifiers

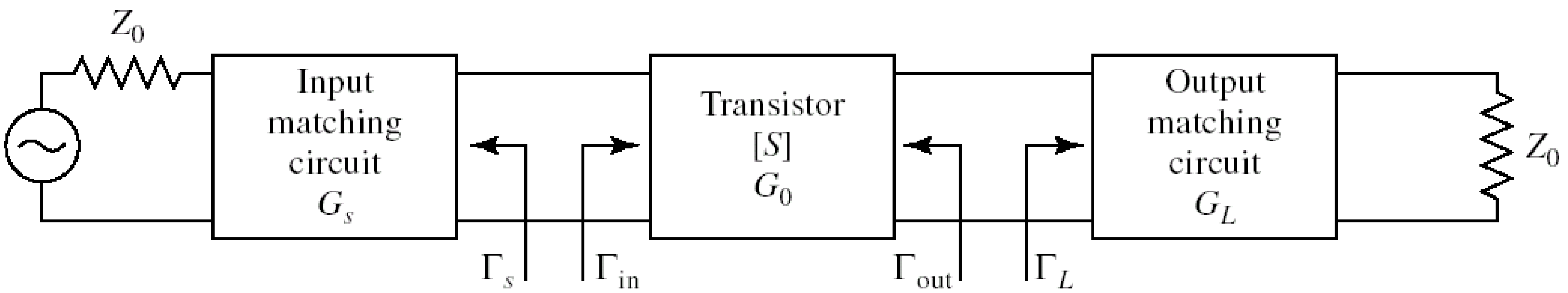
Power Gain of Microwave Amplifiers

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

$$\Gamma_{in} = \Gamma_S^* \qquad \Gamma_{out} = \Gamma_L^*$$

- For lossless matching sections

$$G_{T\max} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2\cdot|1-S_{22}\cdot\Gamma_L|^2} \qquad G_{T\max} = \frac{1}{1-|\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

- For the general case of the bilateral transistor (S12≠0) Γ_{in} and Γ_{out} depend on each other so the input and output sections must be matched simultaneously

Simultaneous matching

$$\Delta \cdot \left(S_{11}^* \cdot S_{22}^* - S_{12}^* \cdot S_{21}^*\right) = |\Delta|^2$$

$$\Gamma_S^2 \cdot \underbrace{\left(S_{11} - \Delta \cdot S_{22}^*\right)}_{C} + \Gamma_S \cdot \underbrace{\left(|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1\right)}_{-B} + \underbrace{\left(S_{11}^* - \Delta^* \cdot S_{22}\right)}_{C^*} = 0$$

- A quadratic equation

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

- Similarly

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

- With variables defined as:

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \quad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

Simultaneous matching

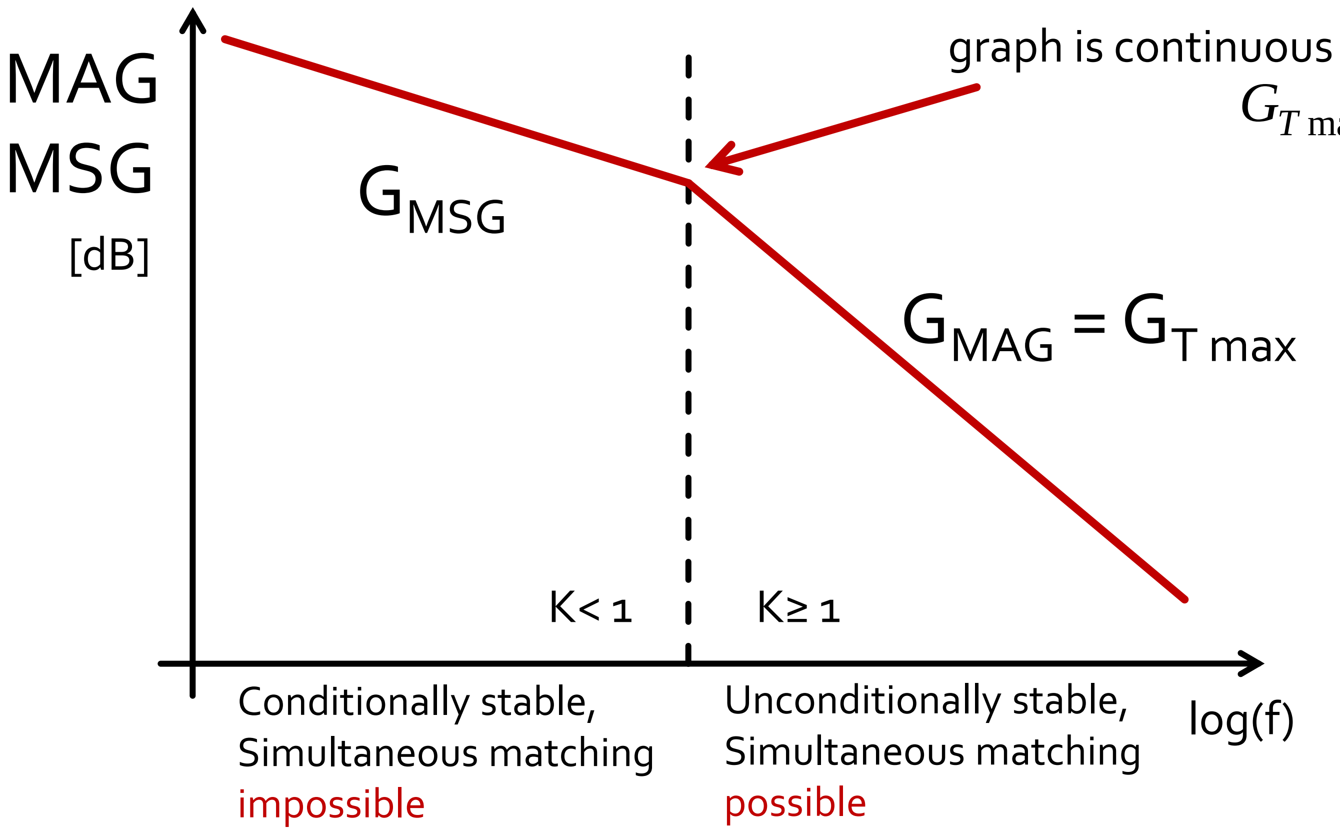
- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and |Γ|<1 solutions are those with "–" sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta\cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta\cdot S_{11}^* \end{cases}$$

Maximum Available Gain

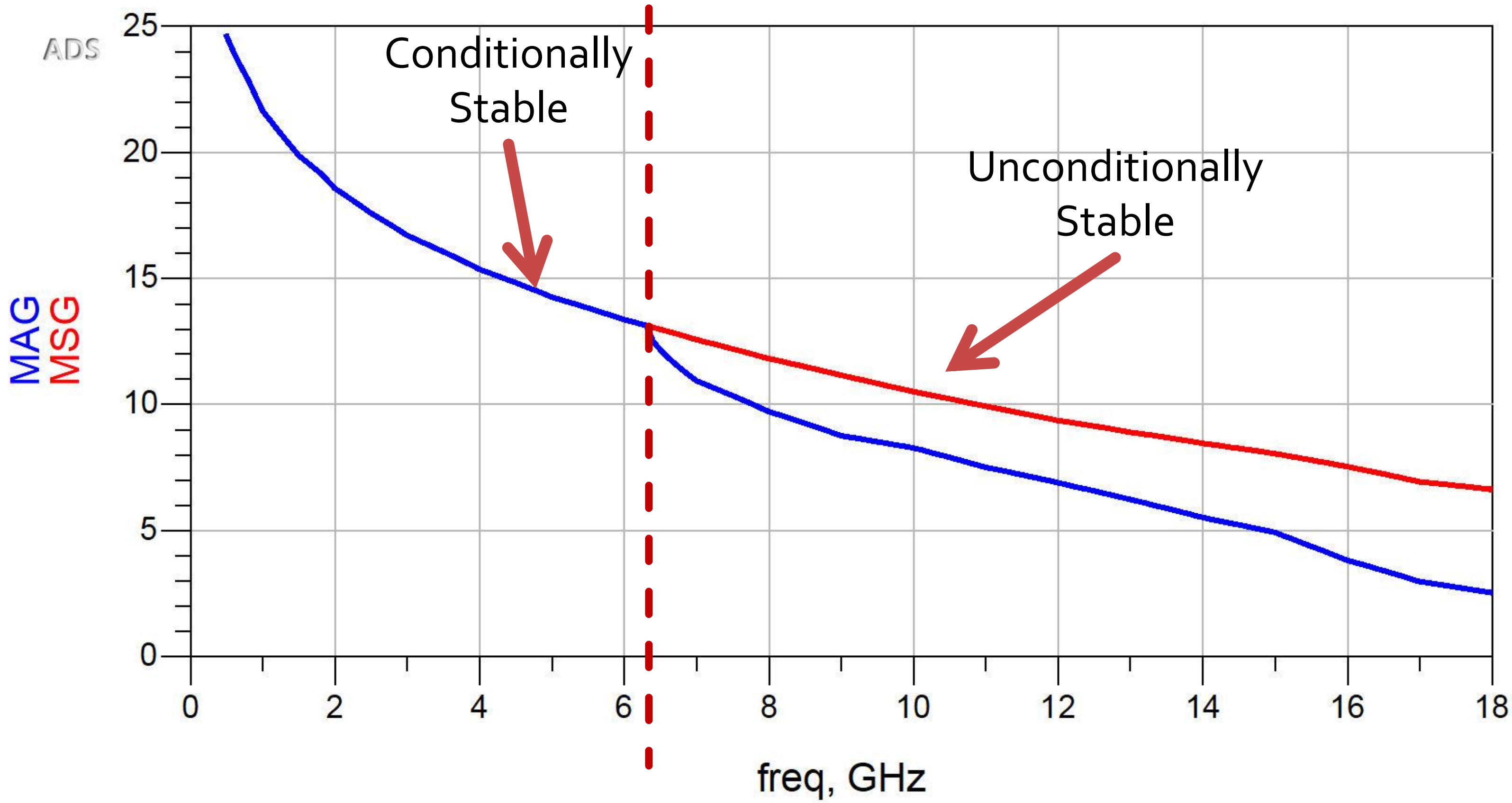
- Indicator across full frequency range of the capability to obtain a power gain



$$G_{T\max}\Big|_{K=1} = \frac{|S_{21}|}{|S_{12}|} = G_{MSG}$$

MAG/MSG

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Microwave Amplifiers

Design for Specified Gain

Design for Specified Gain

- Assumes the amplifier device **unilateral**

Input and output can be treated independently

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Unilateral figure of merit

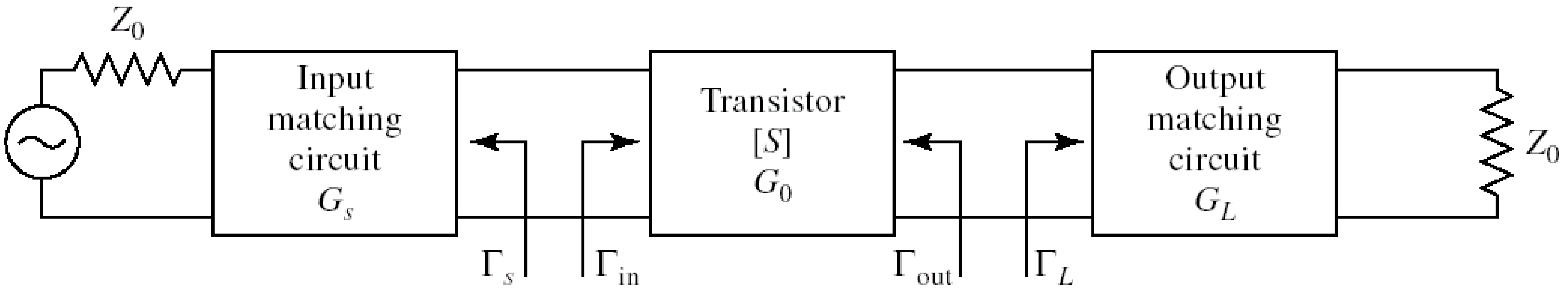
- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \qquad U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1-|S_{11}|^2\right) \cdot \left(1-|S_{22}|^2\right)}$$

- We compute U then the maximum and minimum deviation of G_{TU} from G_T
  - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20 \cdot \log(1+U) < G_T[dB] - G_{TU}[dB] < -20 \cdot \log(1-U)$$

Design for Specified Gain



- In the unilateral assumption:

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

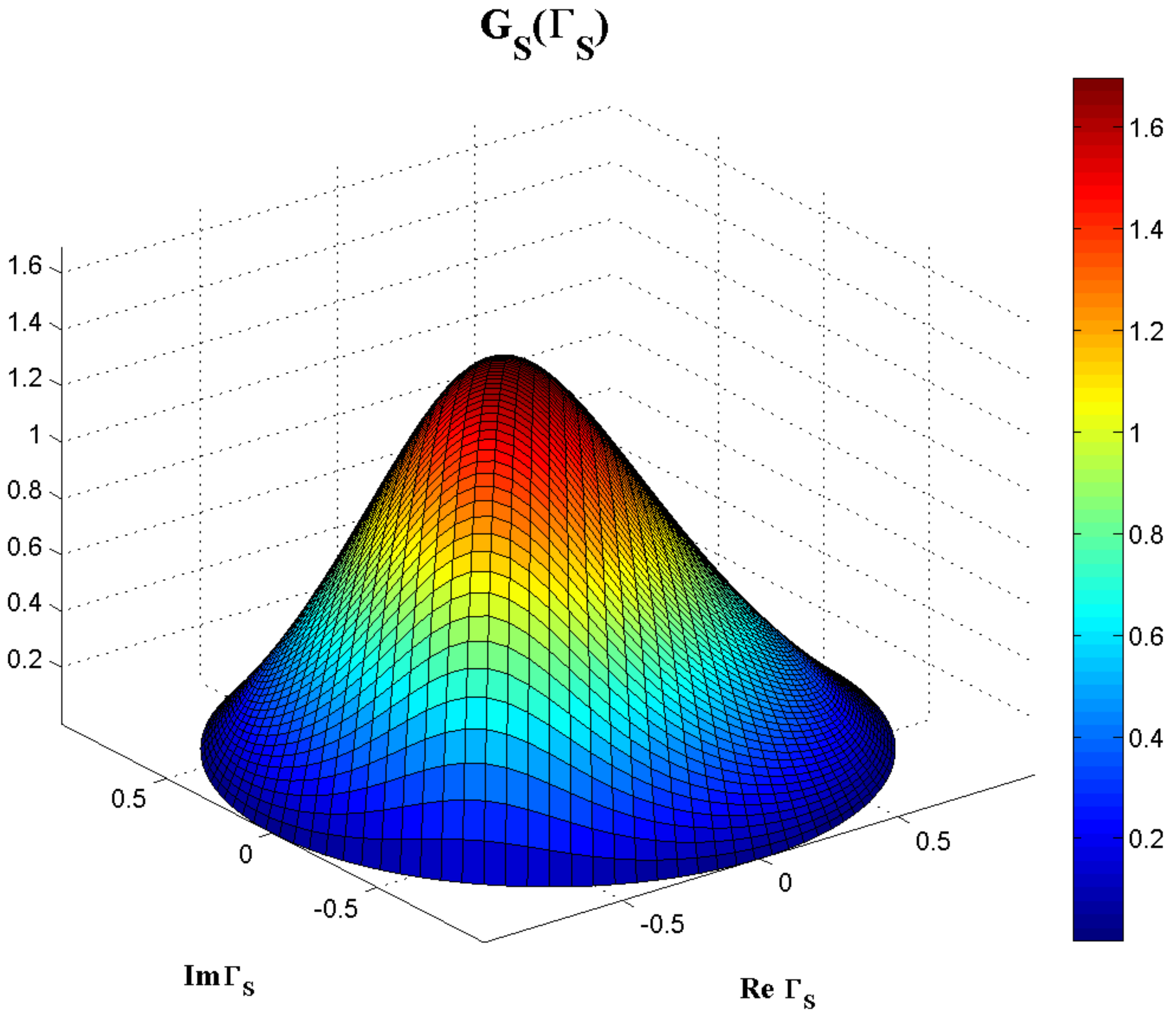
$$G_S = G_S(\Gamma_S)$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

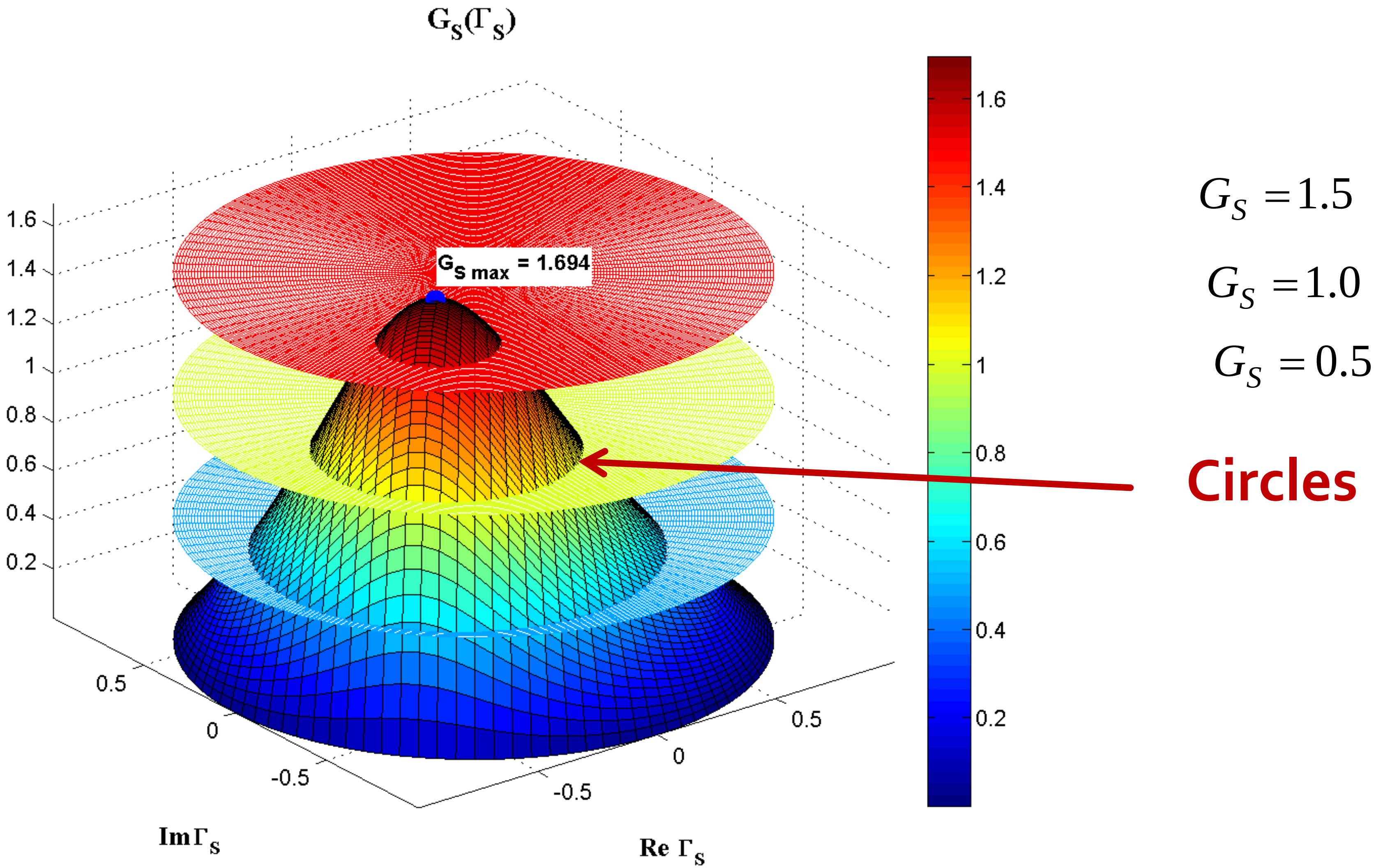
$$G_L = G_L(\Gamma_L)$$

$G_S(\Gamma_S)$

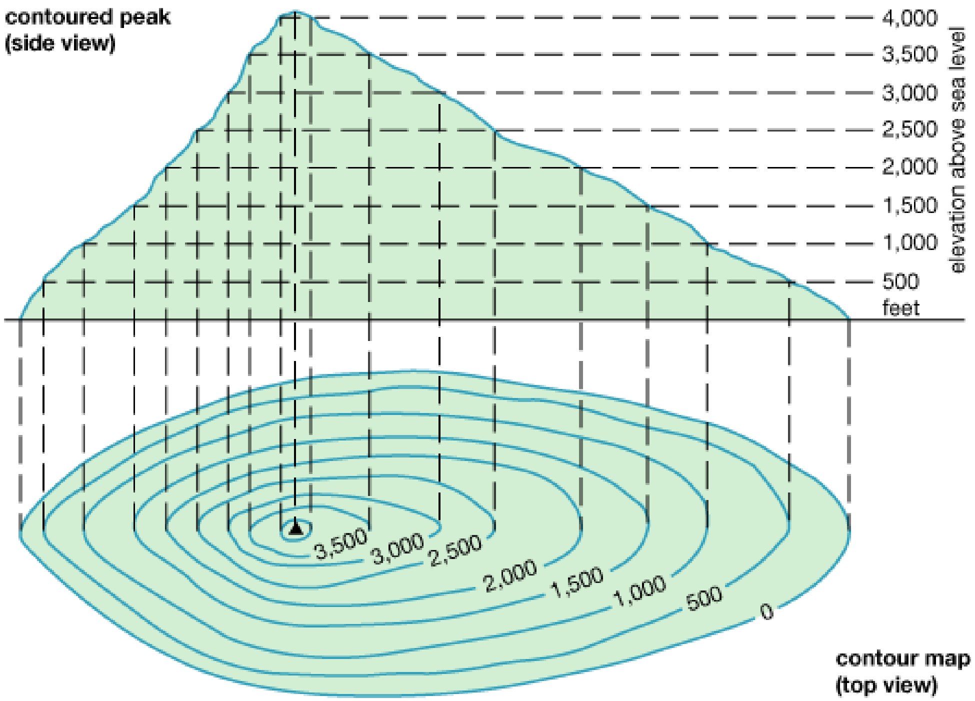


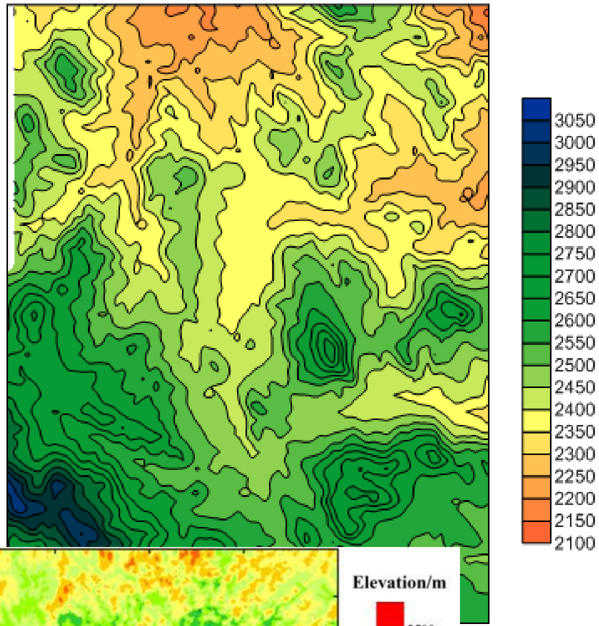
$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}$$

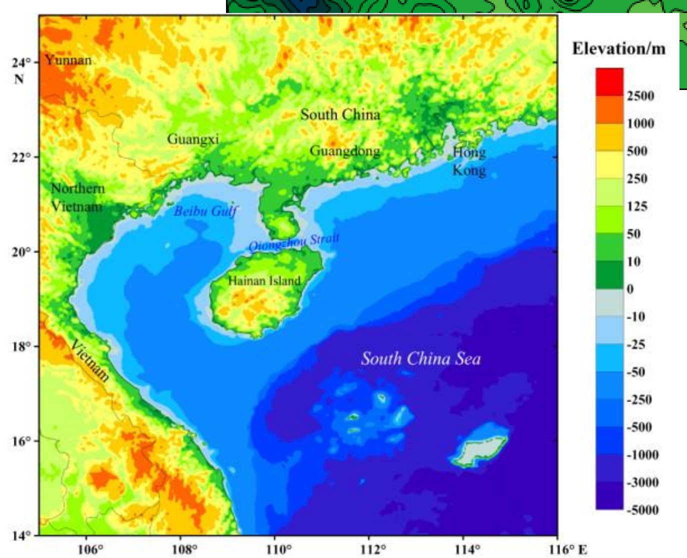
$G_S(\Gamma_S)$, constant value contours



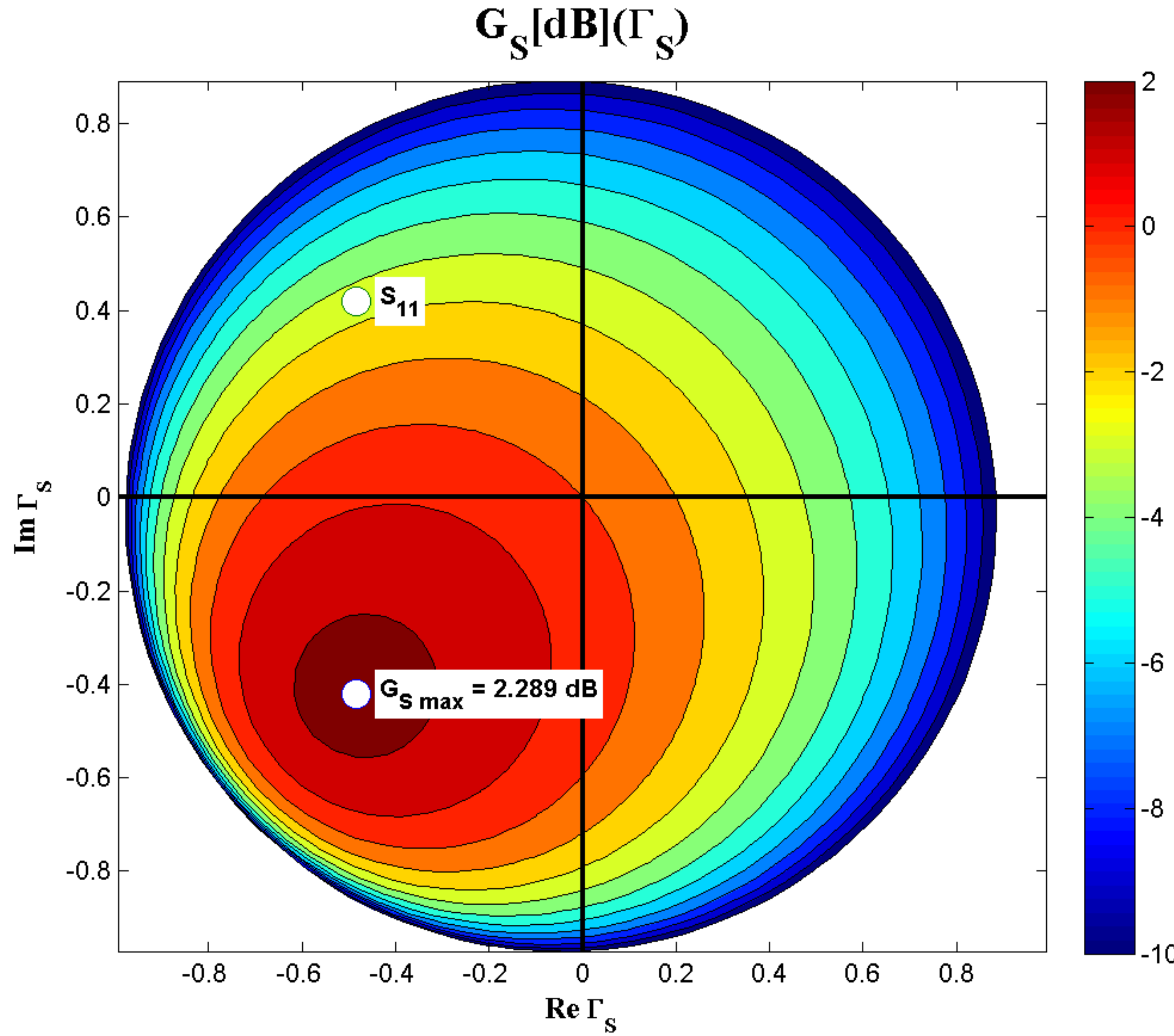
Contour map/lines







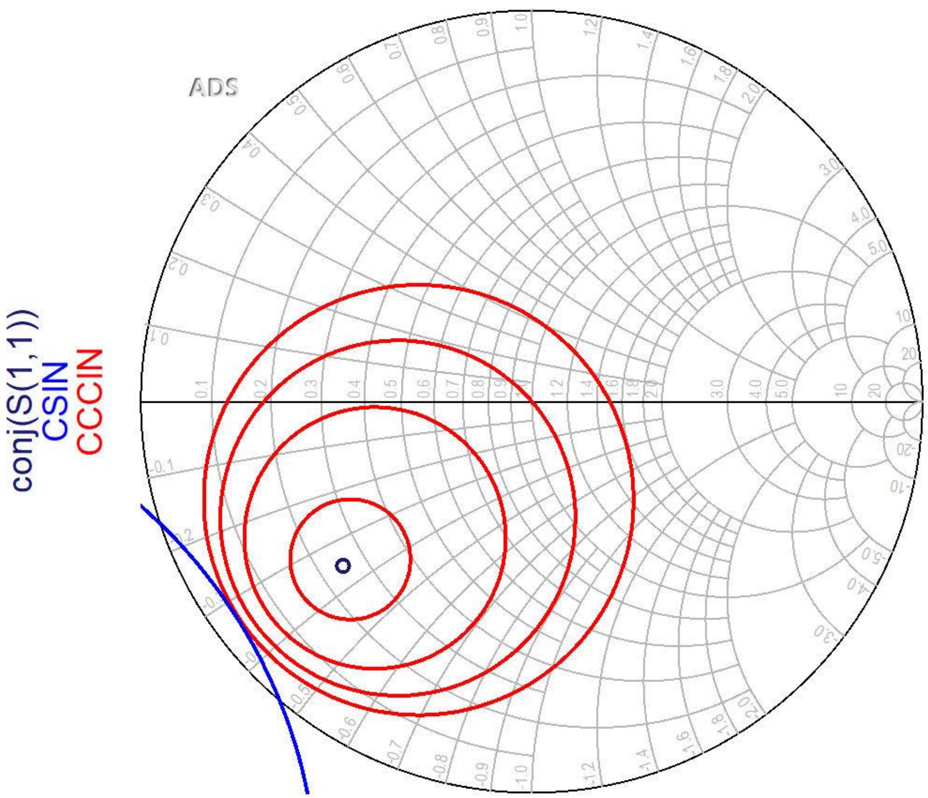
$G_S[dB](\Gamma_S)$, constant value contours

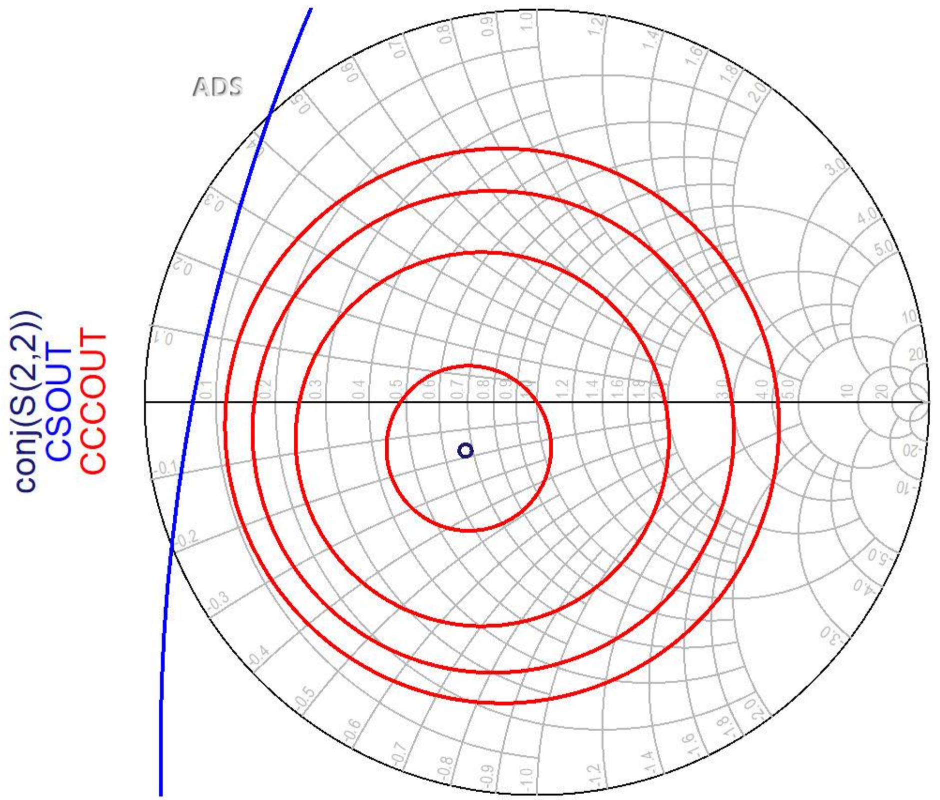


$$G_S[dB] = 10 \cdot \log\left(\frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2}\right)$$

$$G_{S\,\max} = G_S\Big|_{\Gamma_S = S_{11}^*}$$

ADS





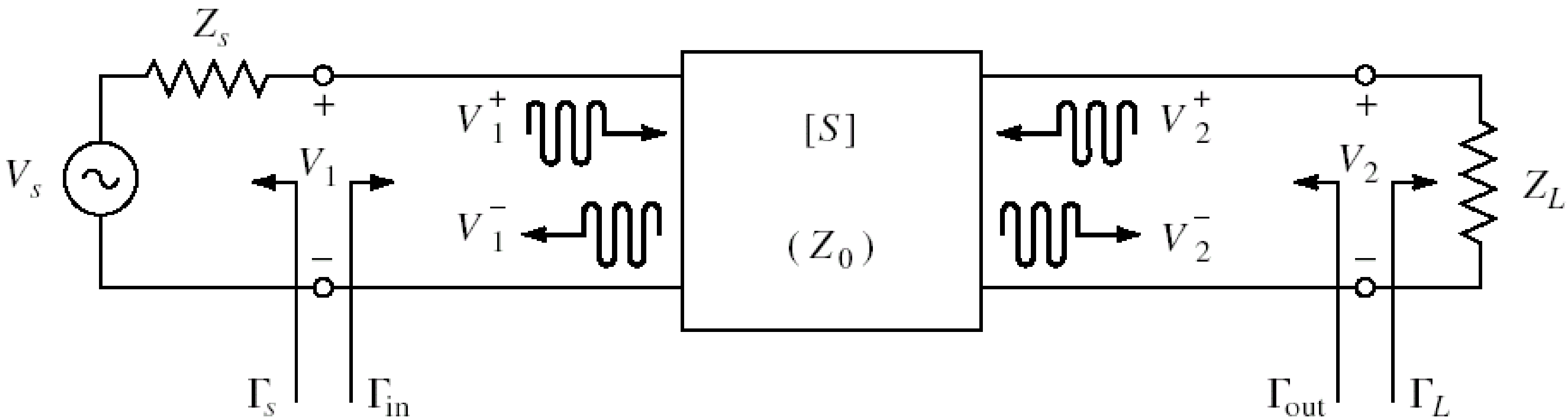
- Circles are plotted for requested values (**in dB!**)
- It is usefull to compute G_{Smax} and G_{Lmax} before
  - in order to request relevant circles

Continue

Microwave Amplifiers

Low-Noise Amplifier Design

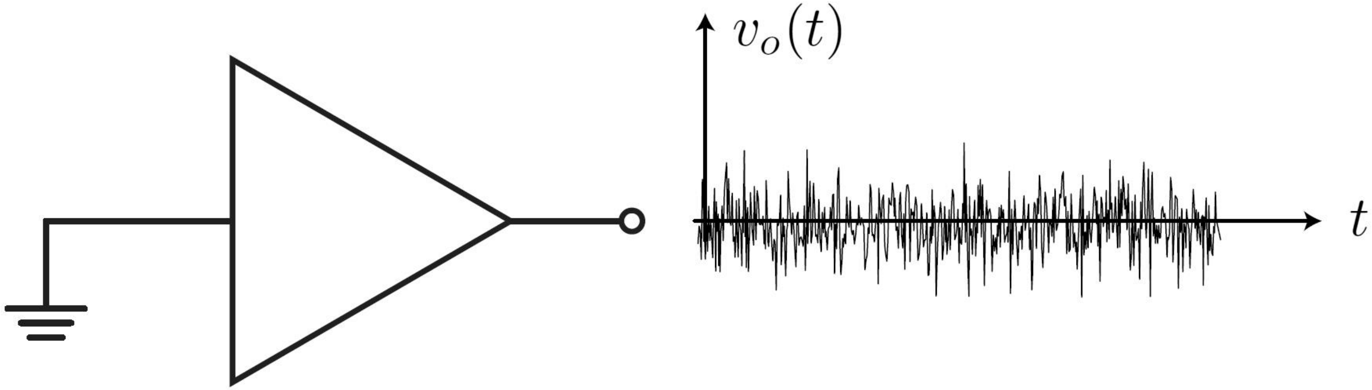
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - **noise** (sometimes – **small signals**)
  - linearity (sometimes – large signals)

Noise

- Noise: random fluctuations of the signal

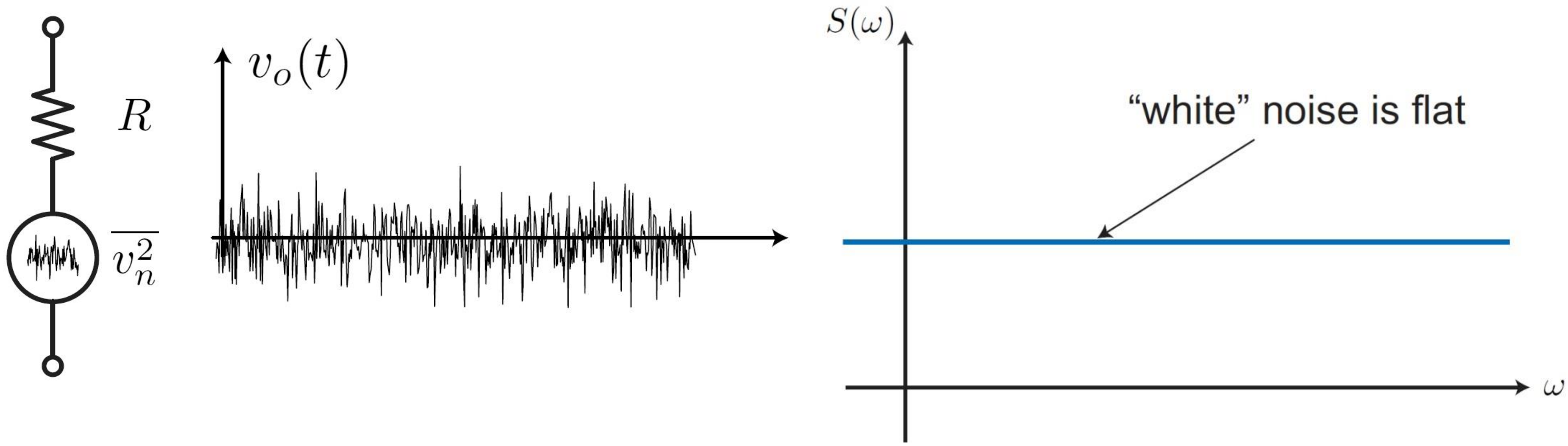


$$\overline{v_n(t)} = \langle v_n(t) \rangle = \frac{1}{T}\int_0^T v_n(t)\,dt = 0$$

$$\overline{v_n^2(t)} = \langle v_n^2(t) \rangle = \frac{1}{T}\int_0^T v_n^2(t)\,dt \neq 0$$

$$V_{n(ef)} = \sqrt{\overline{v_n^2(t)}}$$

Noise



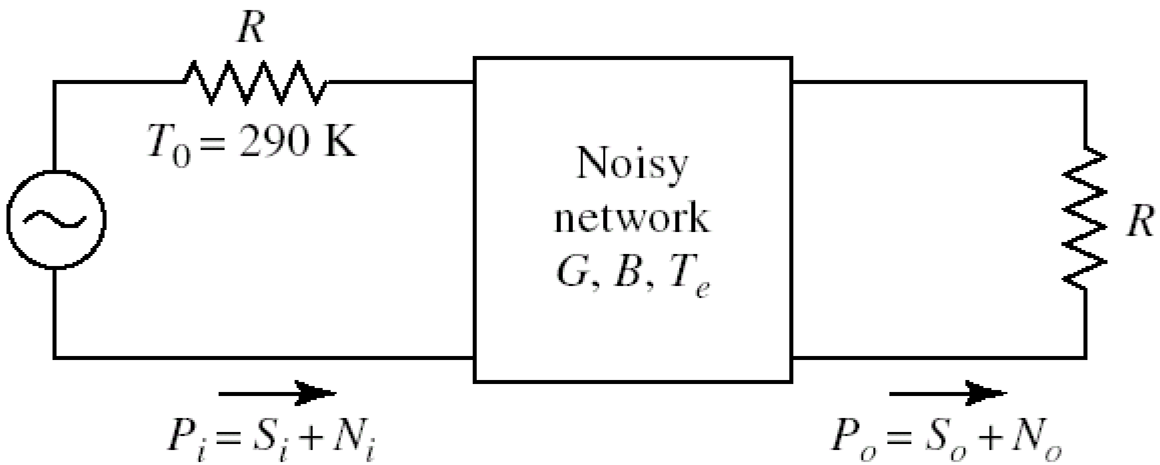
- effective noise voltage

$$V_{n(ef)} = \sqrt{4kTBR}$$

- noise power available (for maximum power transfer with impedance/resistance matching)

$$P_n = kTB$$

Noise Figure F



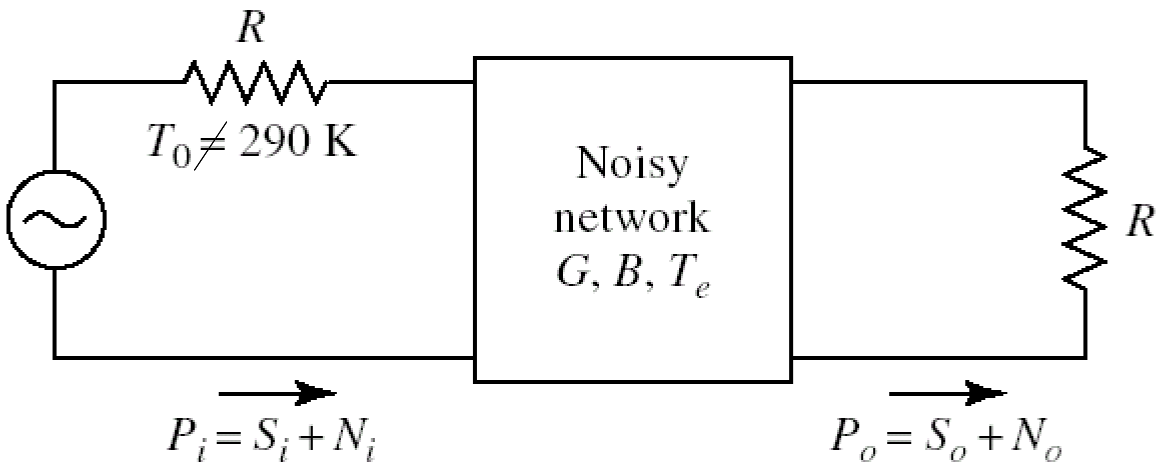
- The noise figure F, is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at To = 290 K (reference noise conditions)

$$F = \left. \frac{S_i / N_i}{S_o / N_o} \right|_{T_0 = 290K}$$

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

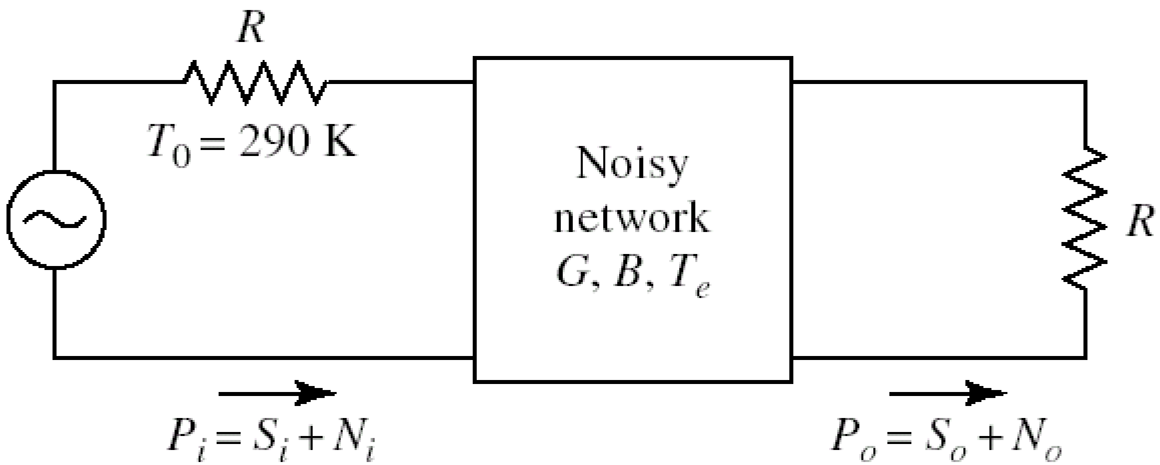
Noise Figure F



- The noise figure F, **is not** directly a measure of the reduction in signal-to-noise ratio between the input and output of a device, when the input noise power is different from that of the reference noise conditions

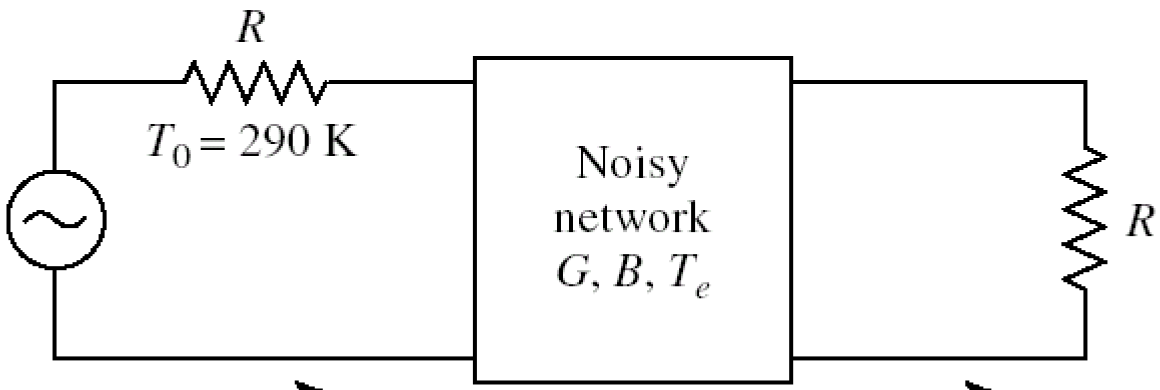
$$F \neq \left. \frac{S_i / N_i}{S_o / N_o} \right|_{T_0 \neq 290 K}$$

Noise Figure F



- In general, the output noise power consists of two elements:
  - the input noise power amplified or attenuated by the device (for example amplified with the power gain G applied also to the desired signal)
  - a noise power generated internally by the network if the network is noisy (this power **does not** depend on the input noise power)

Noise Figure F



$$P_1 = S_1 + N_1 \qquad P_2 = S_2 + N_2$$

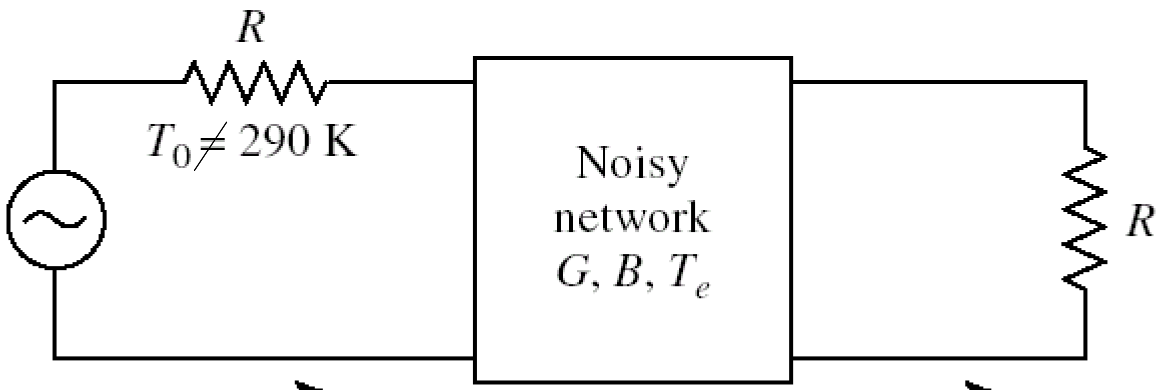
- Estimation of the internally generated noise power can be done using the Noise Figure F definition:

$$F = \left.\frac{S_1/N_1}{S_2/N_2}\right|_{T_0=290K,\, N_1=N_0}$$

$$N_2 = F \cdot N_0 \cdot \frac{S_2}{S_1} = F \cdot N_0 \cdot G$$

$$N_2 = N_0 \cdot G + (F-1) \cdot N_0 \cdot G$$

Noise Figure F



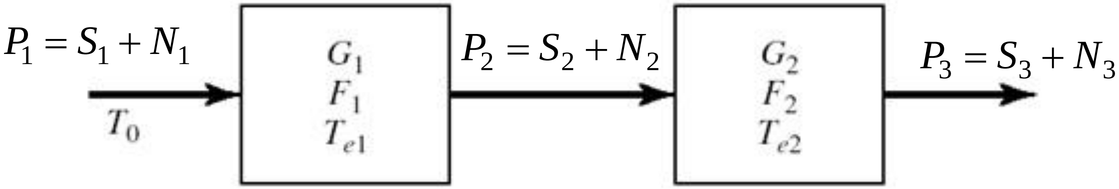
$$P_1 = S_1 + N_1 \qquad P_2 = S_2 + N_2$$

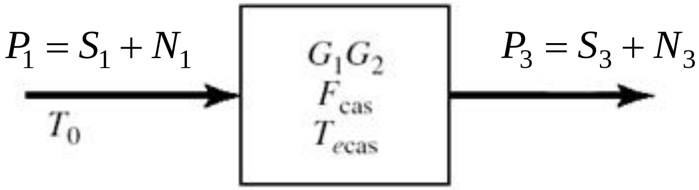
- We identify the two terms:
  - amplified input noise
  - internally generated noise
- When the input noise does not correspond to reference noise conditions (N1 ≠ No)
  - the internally generated noise does not change

$$N_2 = N_0 \cdot G + (F - 1) \cdot N_0 \cdot G$$

$$N_2 = N_1 \cdot G + (F - 1) \cdot N_0 \cdot G$$

Noise figure of a cascaded system





$$N_2 = N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1$$

$$N_3 = N_2 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

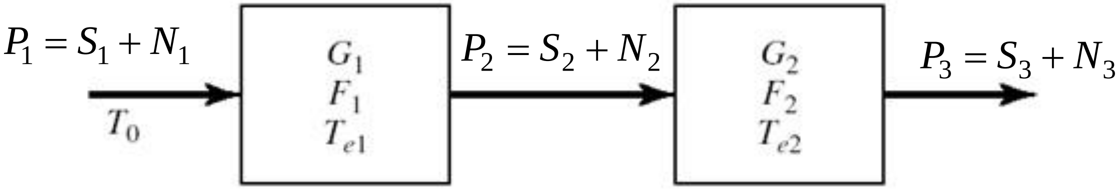
$$G_{cas} = G_1 \cdot G_2$$

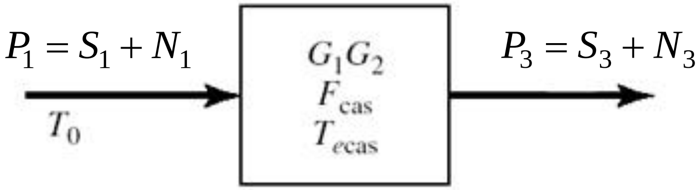
$$N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

$$N_3 = [N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1] \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

Noise figure of a cascaded system





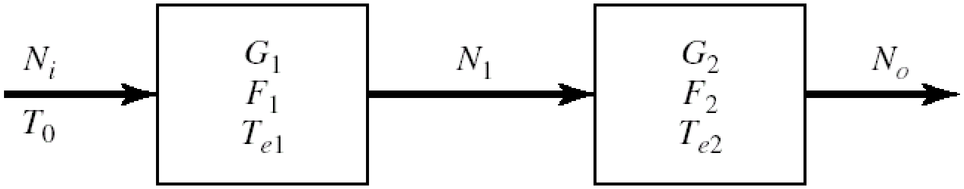
$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2 \qquad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

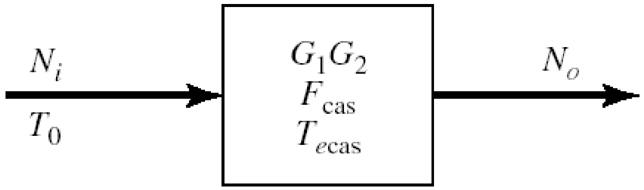
$$(F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 = (F_{cas} - 1) \cdot N_0 \cdot G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

Noise figure of a cascaded system



(a)



(b)

$$G_{cas} = G_1 \cdot G_2 \qquad F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

- Friis Formula (**!linear scale**)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Friis Formula shows that:
  - the overall noise figure of a cascaded system is largely determined by the noise characteristics of the first stage
  - the noise introduced by the following stages is reduced:
    - -1
    - division by G (usually G > 1)

Friis Formula (noise)

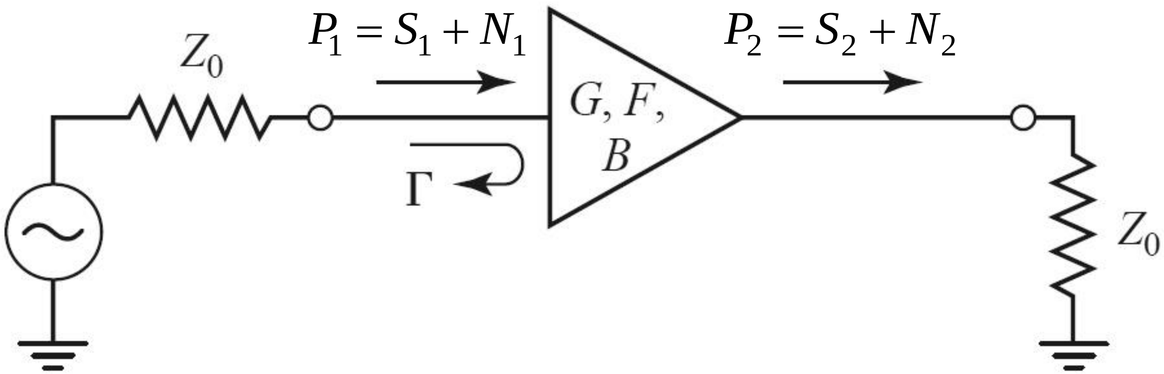
$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Effects of Friis Formula:
- in multi stage amplifiers:
  - it's essential that the first stage is as noiseless as possible even if that means sacrificing power gain
  - the following stages can be optimized for power gain
- in single stage amplifiers:
  - in the input matching circuit it's important to have noiseless elements (pure reactance, lossless lines)
  - output matching circuit has less influence on the noise (noise generated at this level appears when the desired signal has already been amplified by the transistor)

$$V_{n(ef)} = \sqrt{4kTBR} \qquad P_n = kTB$$

Noise Figure of a Mismatched Amplifier

- An input mismatched amplifier($\Gamma \neq 0$)



$$G_{ech} = G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + (F - 1) \cdot N_0 \cdot G = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + \frac{F-1}{1-|\Gamma|^2} \cdot N_0 \cdot G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G_{ech} + (F_{ech} - 1) \cdot N_0 \cdot G_{ech}$$

$$F_{ech} = 1 + \frac{F-1}{1-|\Gamma|^2} \geq F$$

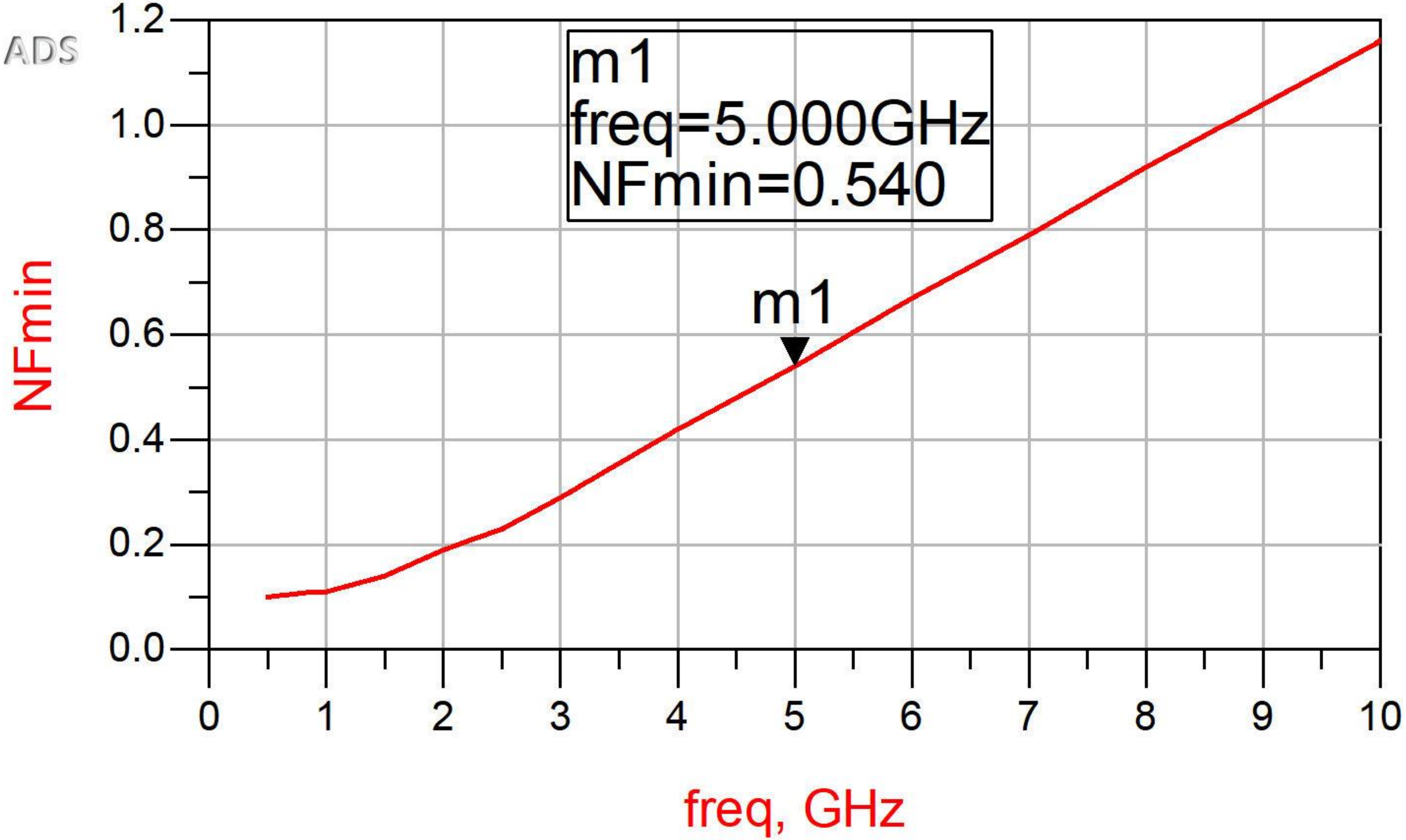
- Good noise figure **requires** good impedance matching

Example

- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - $S_{11} = 0.64\angle 139°$
  - $S_{12} = 0.119\angle -21°$
  - $S_{21} = 3.165\angle 16°$
  - $S_{22} = 0.22\angle 146°$
  - Fmin = 0.54 (**tipic [dB]**)
  - $\Gamma_{opt} = 0.45\angle 174°$
  - $r_n = 0.03$

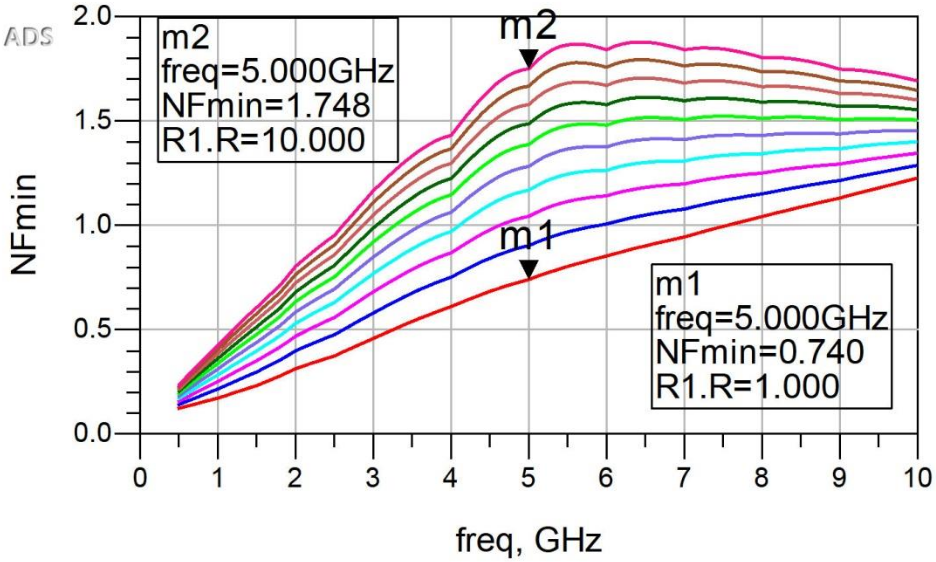
```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt    GAMMA OPT      RN/Zo
!GHZ     dB     MAG     ANG       -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

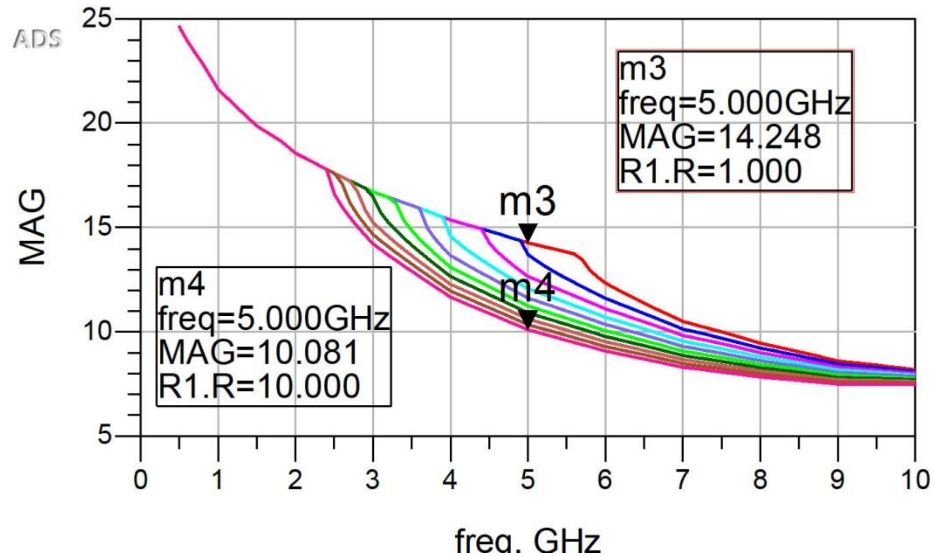
Example



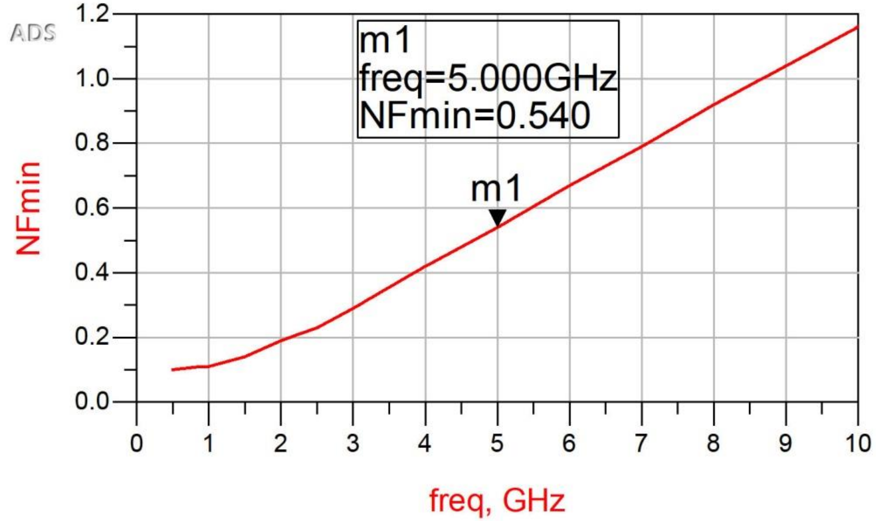
ADS
m1
freq=5.000GHz
NFmin=0.540
m1
NFmin
freq, GHz
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
9
10

Stabilization, input series resistor

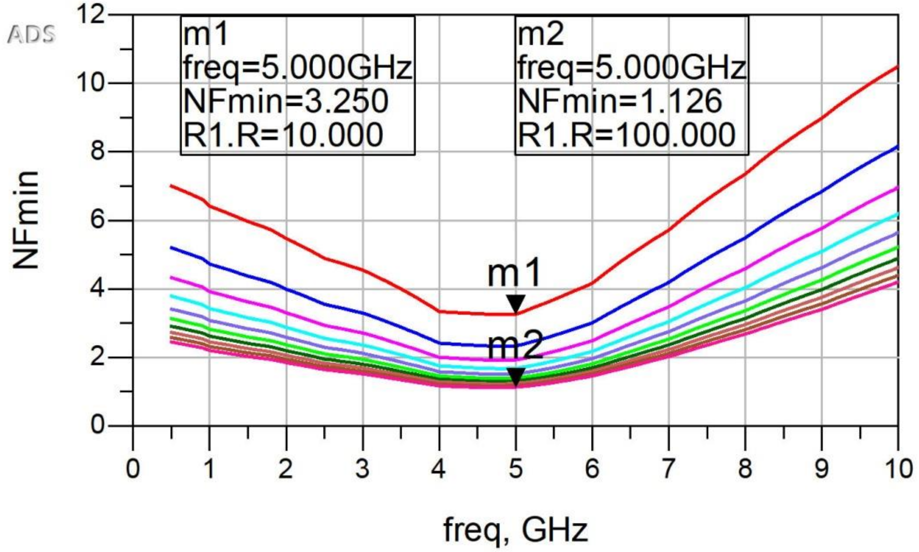


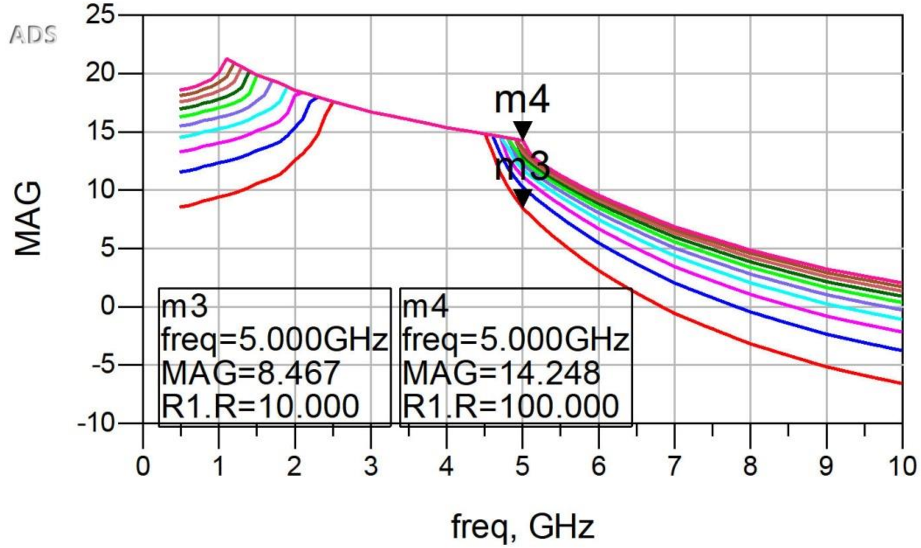


$$R_{SS} = 1 \div 10\,\Omega$$

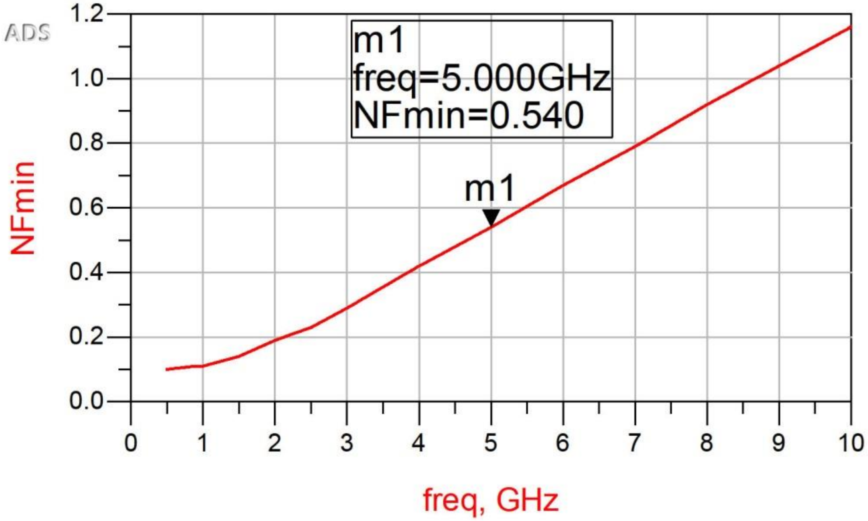


Stabilization, input shunt resistor

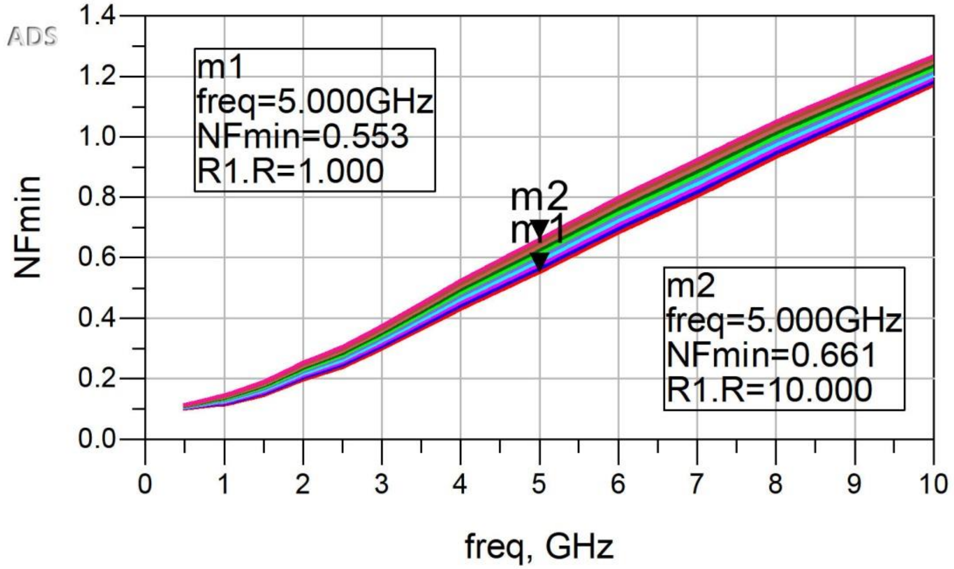


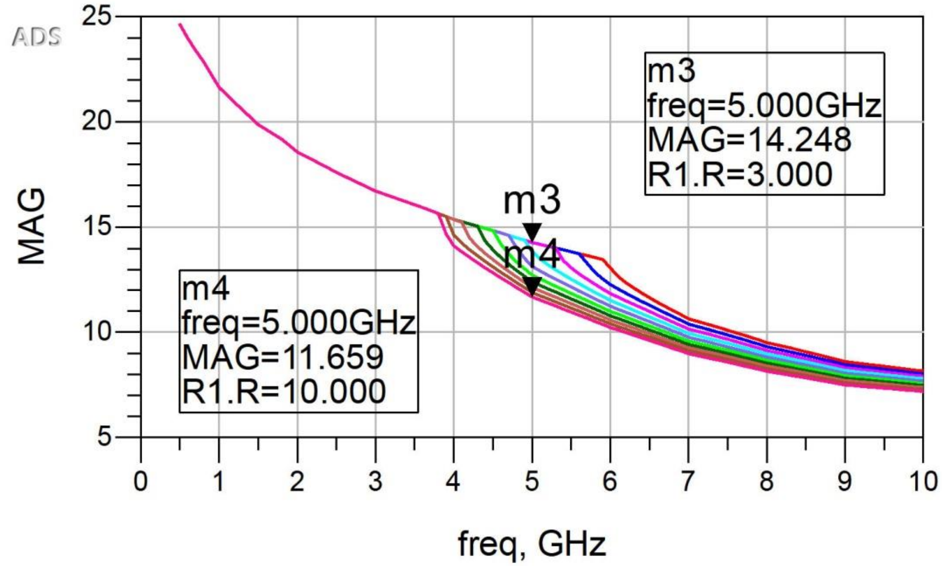


$$R_{PS} = 10 \div 100\,\Omega$$

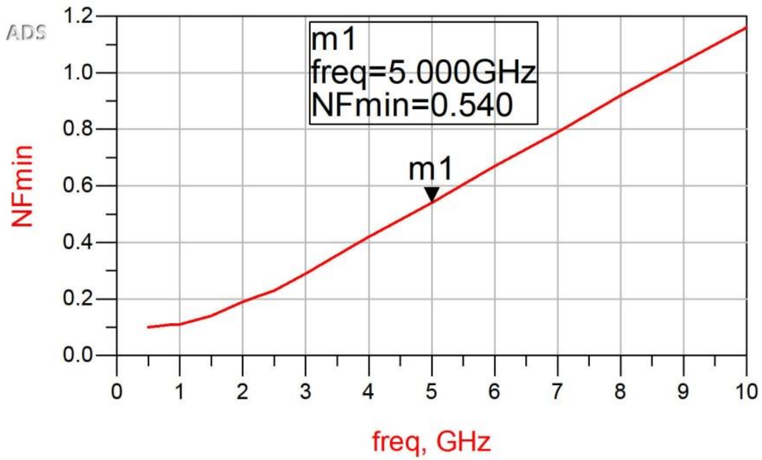


Stabilization, output series resistor

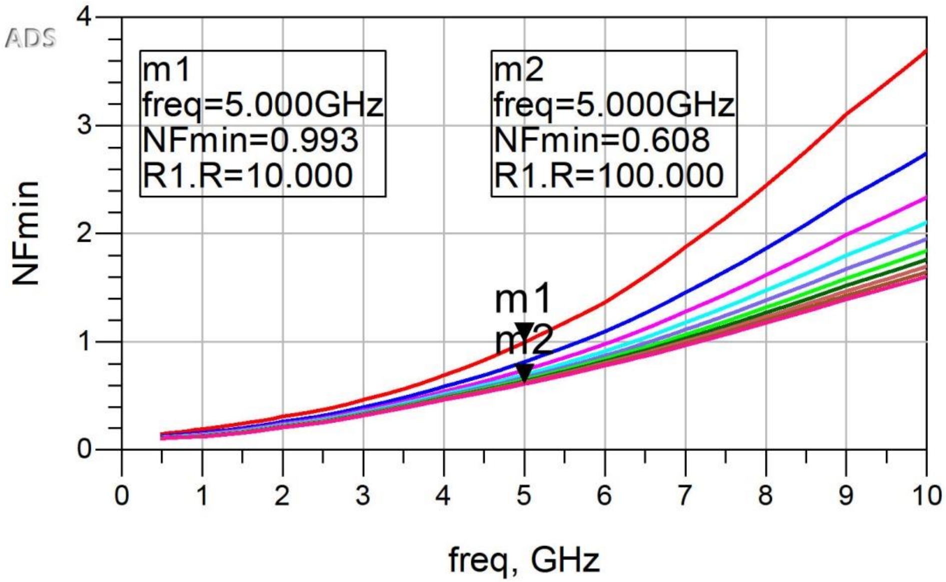


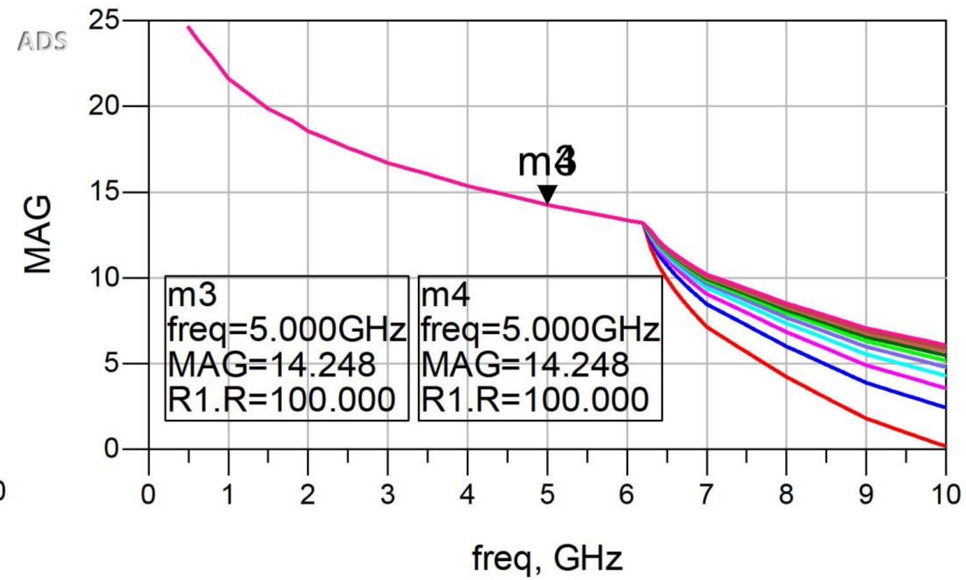


$$R_{SL} = 1 \div 10\,\Omega$$

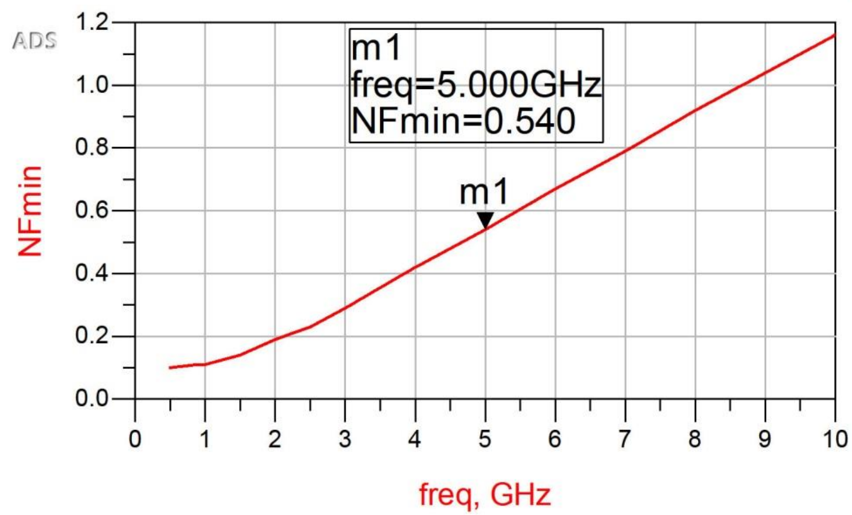


Stabilization, output shunt resistor





$$R_{PL} = 10 \div 100\,\Omega$$



Noise figure of a two-port amplifier

- 3 noise parameters (2reals + 1 complex):

$$F_{\min},\ r_n = \frac{R_N}{Z_0},\ \Gamma_{opt}$$

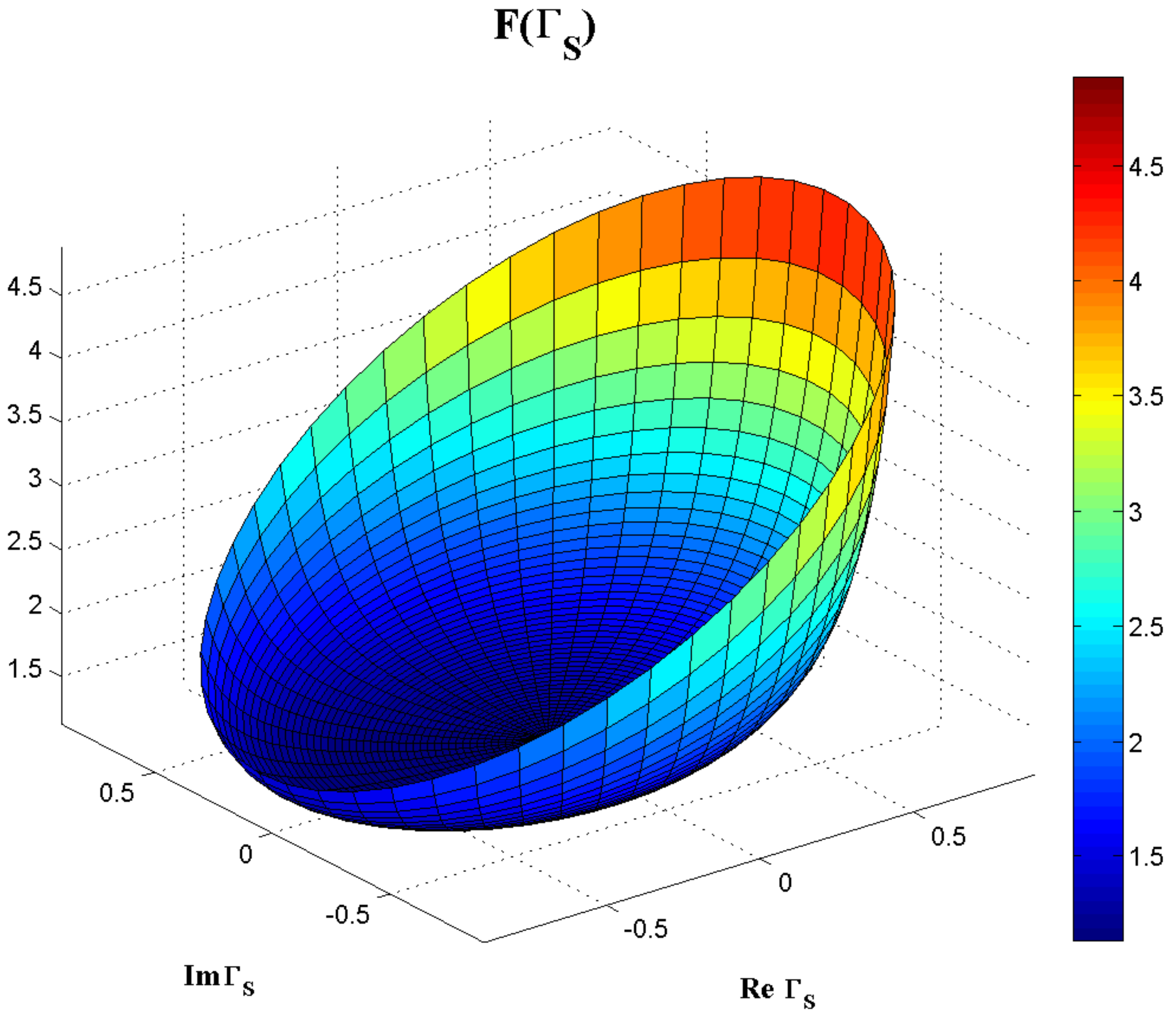
$$\boxed{F = F_{\min} + \frac{R_N}{G_S} \cdot \left|Y_S - Y_{opt}\right|^2} \qquad Y_S = \frac{1}{Z_0} \cdot \frac{1-\Gamma_S}{1+\Gamma_S} \qquad Y_{opt} = \frac{1}{Z_0} \cdot \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1-\left|\Gamma_S\right|^2\right) \cdot \left|1+\Gamma_{opt}\right|^2}$$

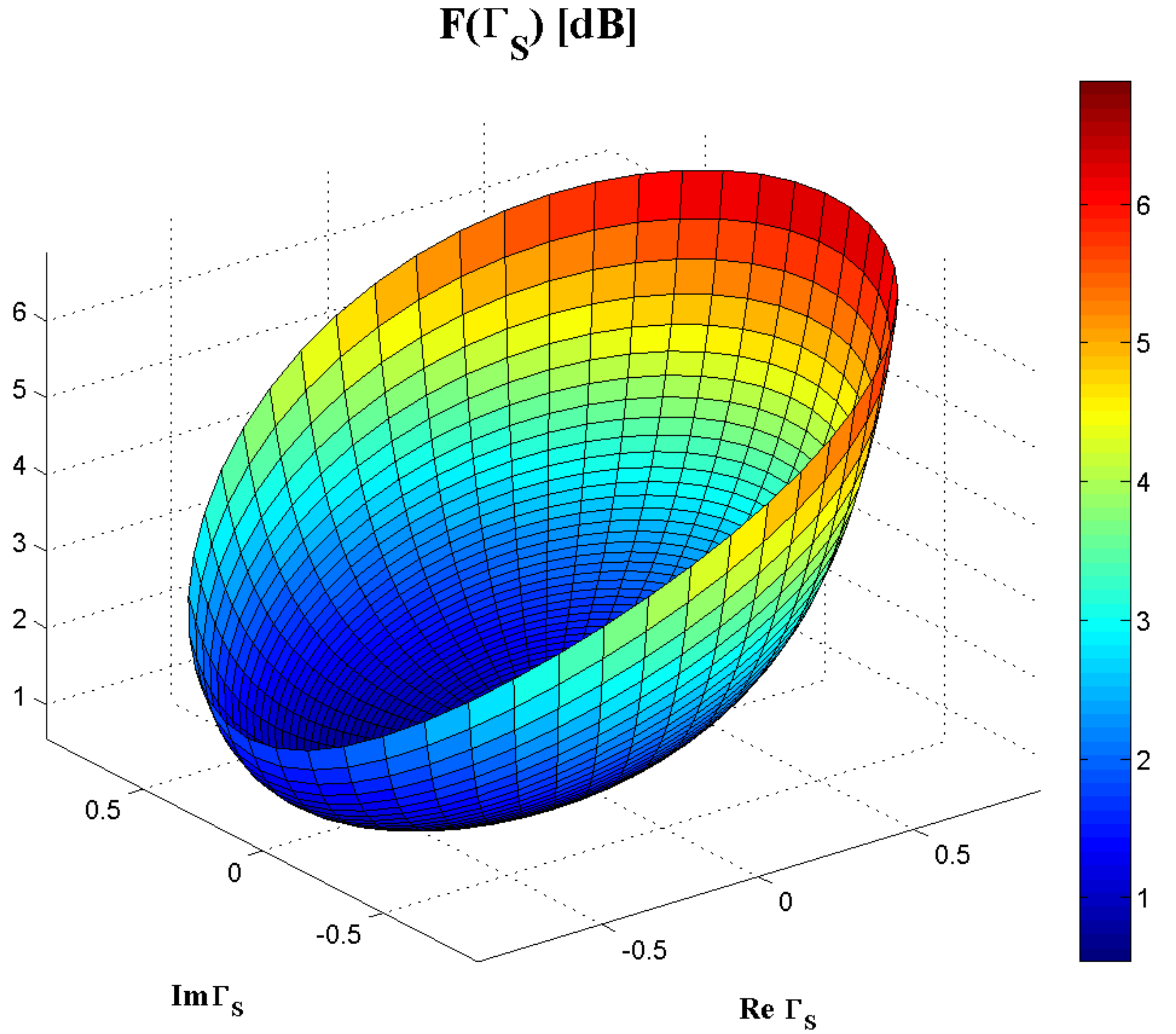
- Γ_{opt} optimum source reflection coefficient that results in minimum noise figure

$$\Gamma_S = \Gamma_{opt} \Rightarrow F = F_{\min}$$

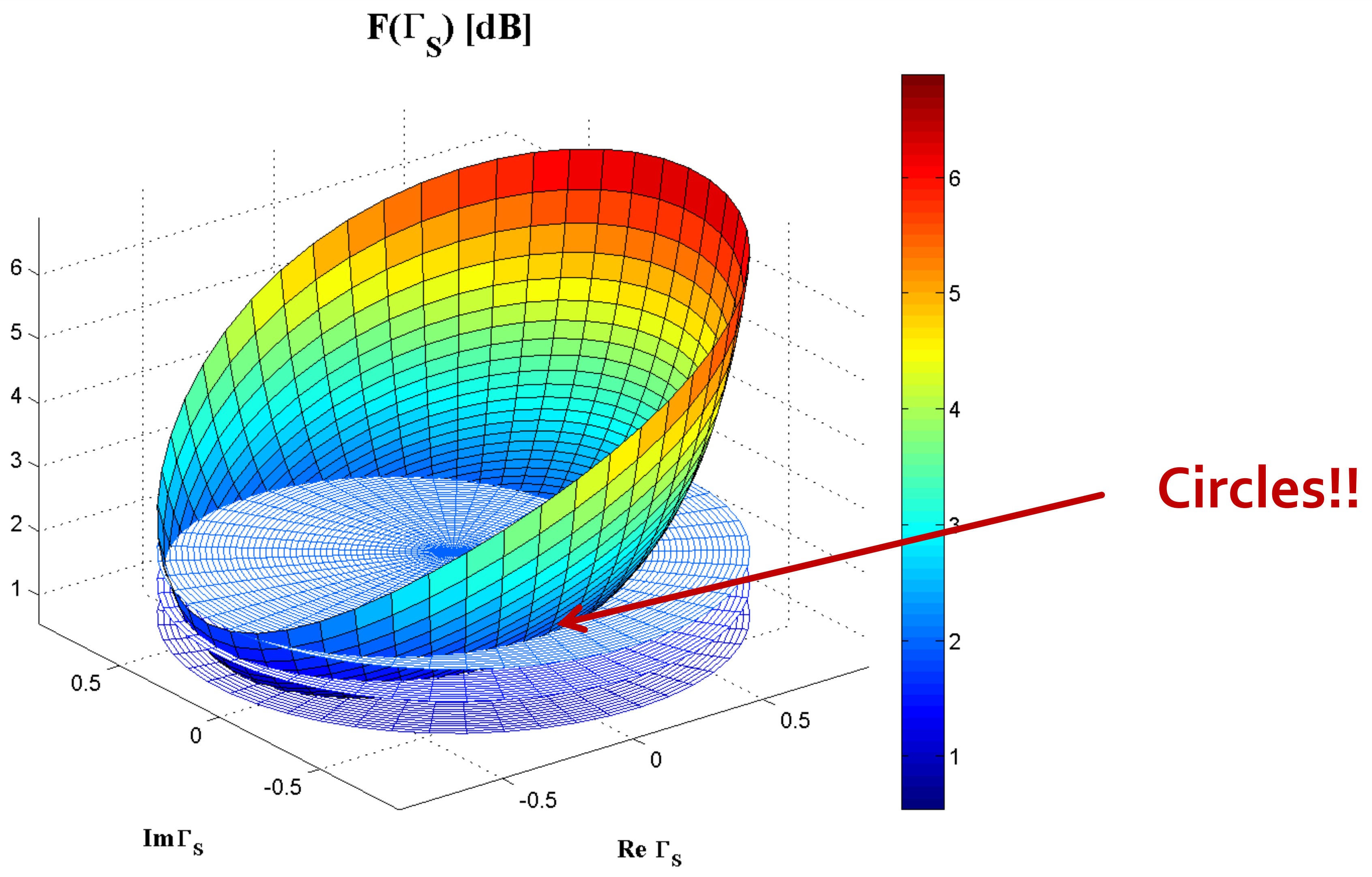
$F(\Gamma_S)$



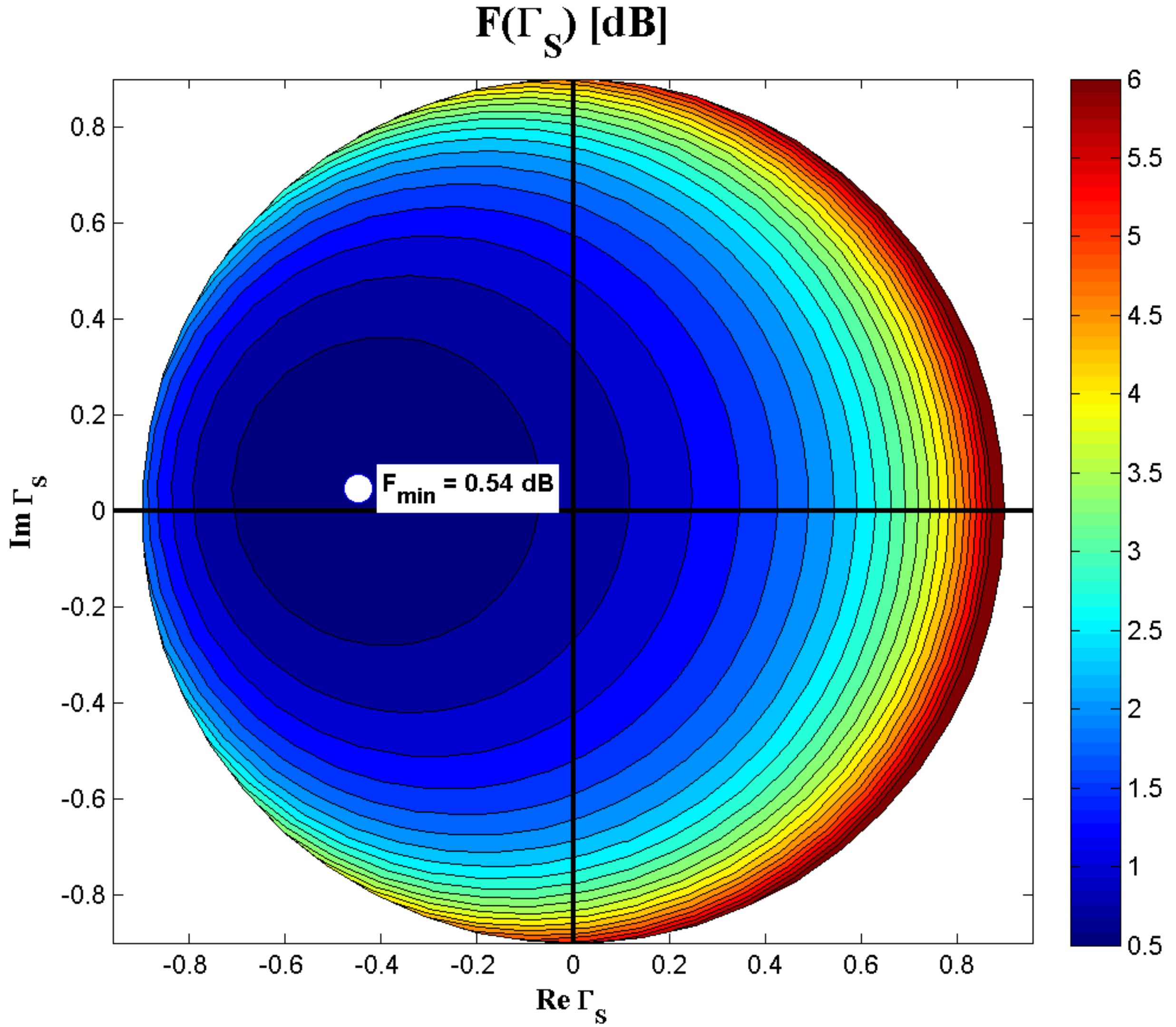
F[dB](Γ_S)



F[dB](Γ_S), constant value contours



$G_S[dB](\Gamma_S)$, constant value contours



$\Gamma opt = 0.45 \angle 174°$

Circles of constant noise figure

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1 - \left|\Gamma_S\right|^2\right) \cdot \left|1 + \Gamma_{opt}\right|^2}$$

- We define N (noise figure parameter)
  - **N** constant for **F** constant

$$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1 - \left|\Gamma_S\right|^2} = \frac{F - F_{\min}}{4 \cdot r_n} \cdot \left|1 + \Gamma_{opt}\right|^2$$

$$\left(\Gamma_S - \Gamma_{opt}\right) \cdot \left(\Gamma_S^* - \Gamma_{opt}^*\right) = N \cdot \left(1 - \left|\Gamma_S\right|^2\right)$$

$$\Gamma_S \cdot \Gamma_S^* + N \cdot \left|\Gamma_S\right|^2 - \left(\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}\right) + \Gamma_{opt} \cdot \Gamma_{opt}^* = N$$

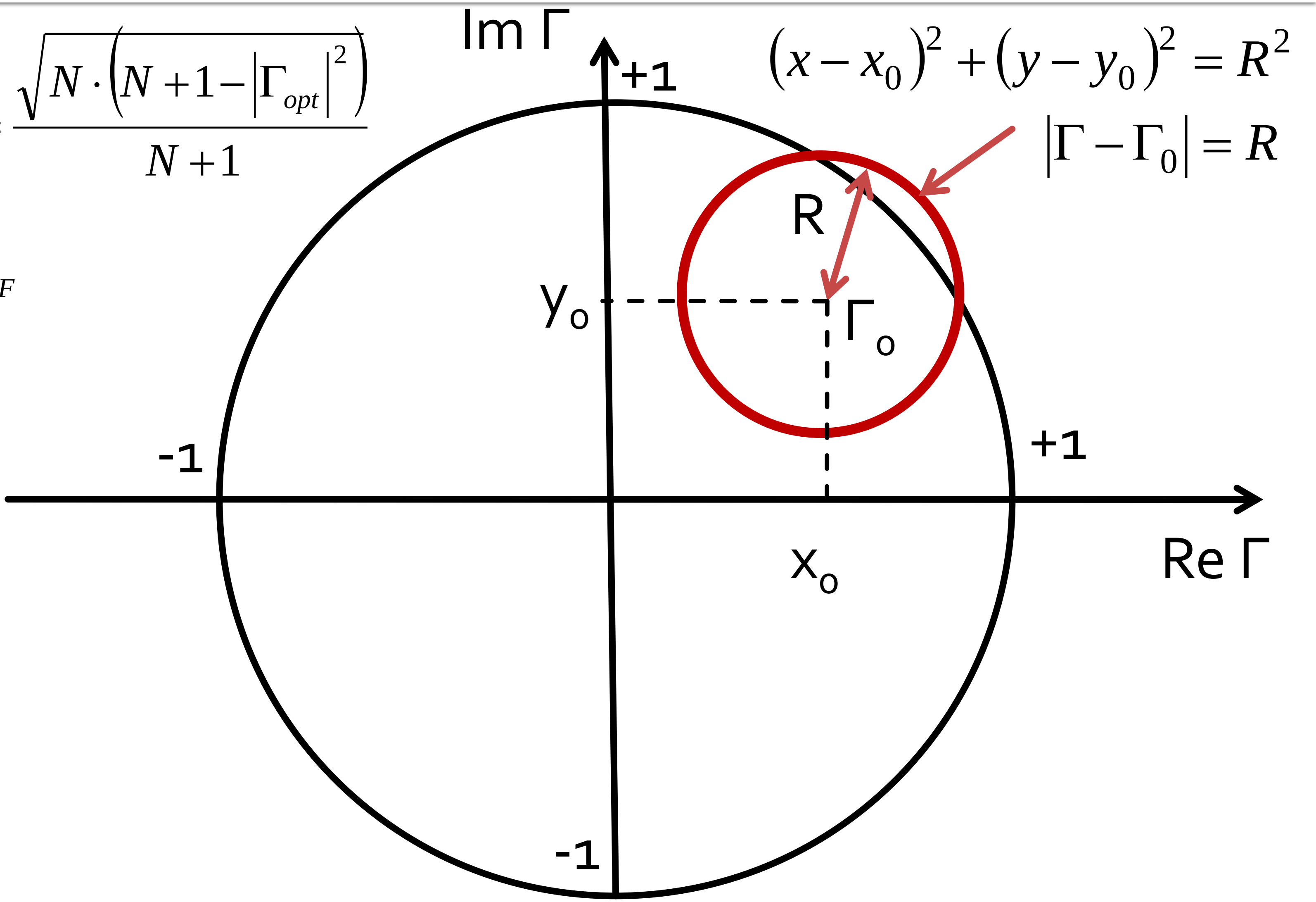
$$\Gamma_S \cdot \Gamma_S^* - \frac{\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}}{N + 1} + \Gamma_{opt} \cdot \Gamma_{opt}^* = \frac{N - \left|\Gamma_{opt}\right|^2}{N + 1} + \frac{\left|\Gamma_{opt}\right|^2}{(N + 1)^2}$$

$$\left|a + b\right|^2 = (a + b) \cdot (a + b)^* = (a + b) \cdot \left(a^* + b^*\right) = \left|a\right|^2 + \left|b\right|^2 + a^* \cdot b + a \cdot b^*$$

Circles of constant noise figure

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N\cdot\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

$$\left|\Gamma_S - C_F\right| = R_F$$



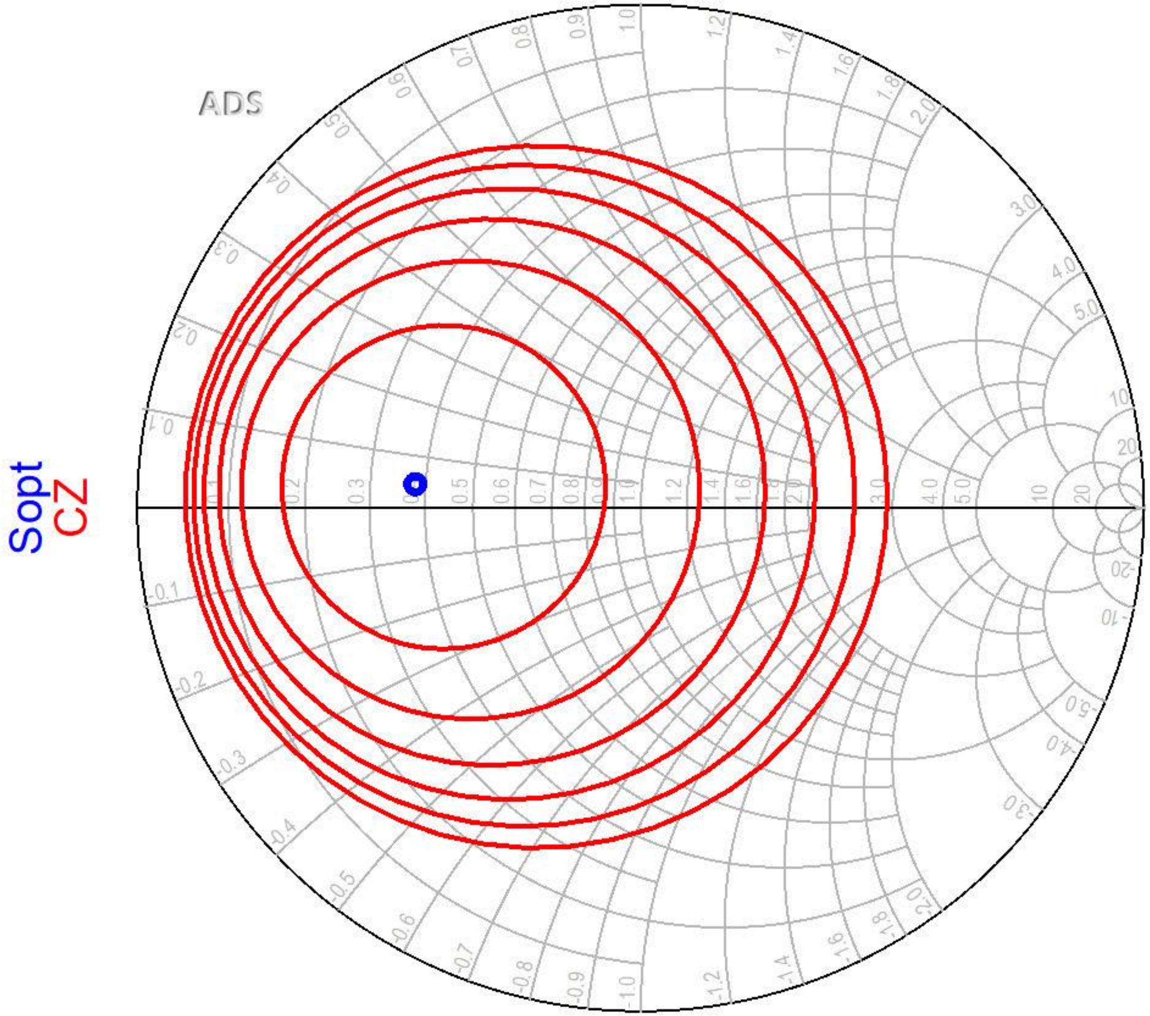
Circles of constant noise figure

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N\cdot\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1} \qquad N = \frac{F-F_{\min}}{4\cdot r_n}\cdot\left|1+\Gamma_{opt}\right|^2$$

$$\left|\Gamma_S - C_F\right| = R_F \qquad C_F = \frac{\Gamma_{opt}}{N+1} \qquad R_F = \frac{\sqrt{N\cdot\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

- The locus in the complex plane Γ_S of the points with constant noise figure is a circle
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for F_{circle} will lead to a noise factor $F = F_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a noise factor $F > F_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a noise factor $F < F_{circle}$

ADS

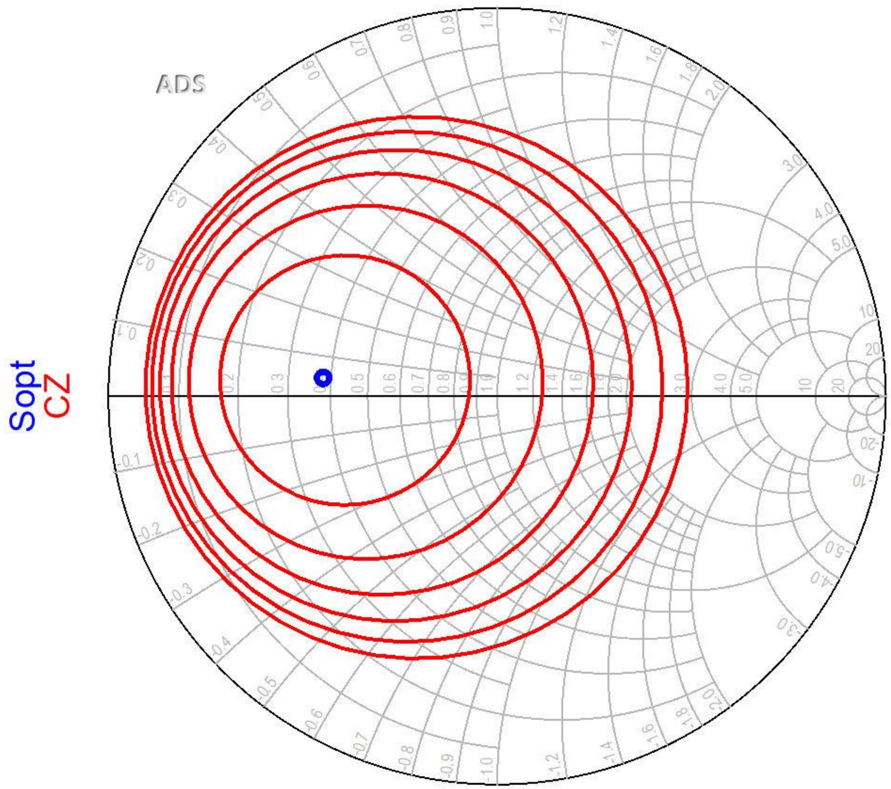


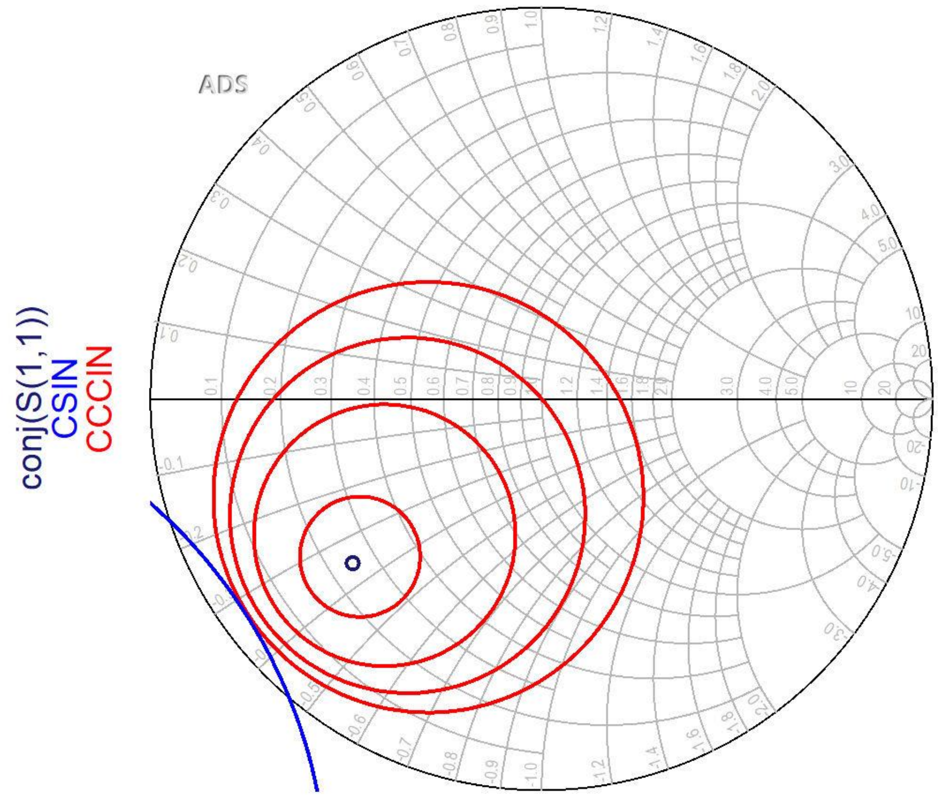
Circles of constant noise figure

- The noise internally generated by the transistor depends **only** by the input matching circuit
- A minimum noise figure is possible (NF_{min} – a datasheet/"s2p file" parameter for the transistor)
- If we design a low noise amplifier (**LNA**) the usual design technique is as follows:
  - design of the input matching circuit solely (largely) for noise optimization
  - design of output matching circuit for gain compensation/optimization (if lossy circuits are used the output matching circuit noise can be added but the transistor noise is not influenced)

LNA – Low Noise Amplifier

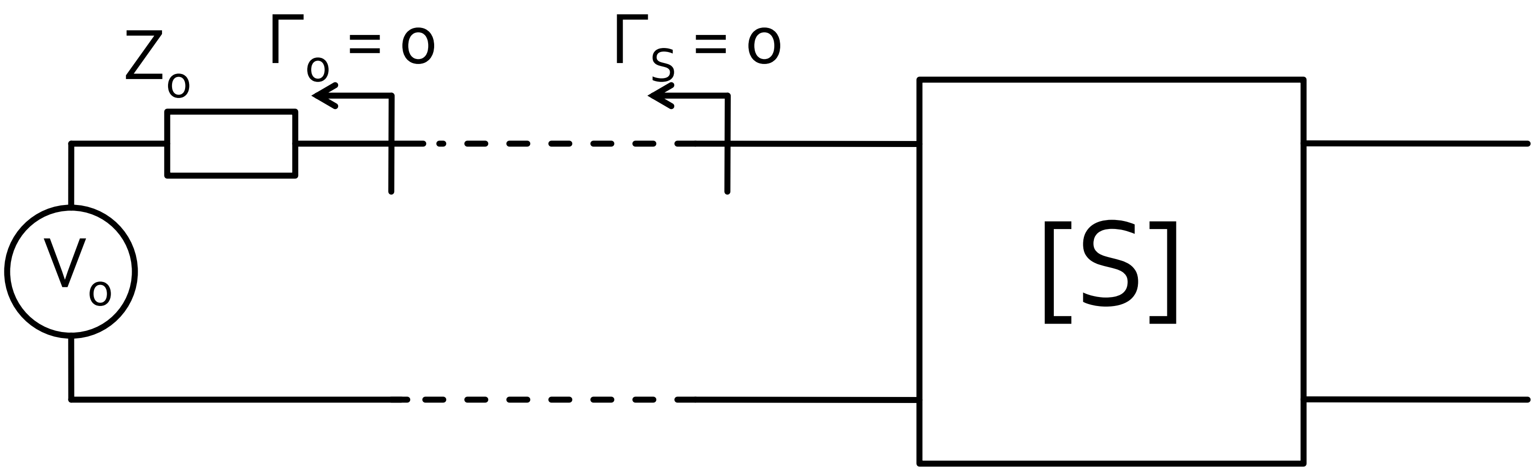
- Usually a transistor suitable for implementing an LNA at a certain frequency will have input gain circles and noise circles in the same area for Γ_S





Matching – 1

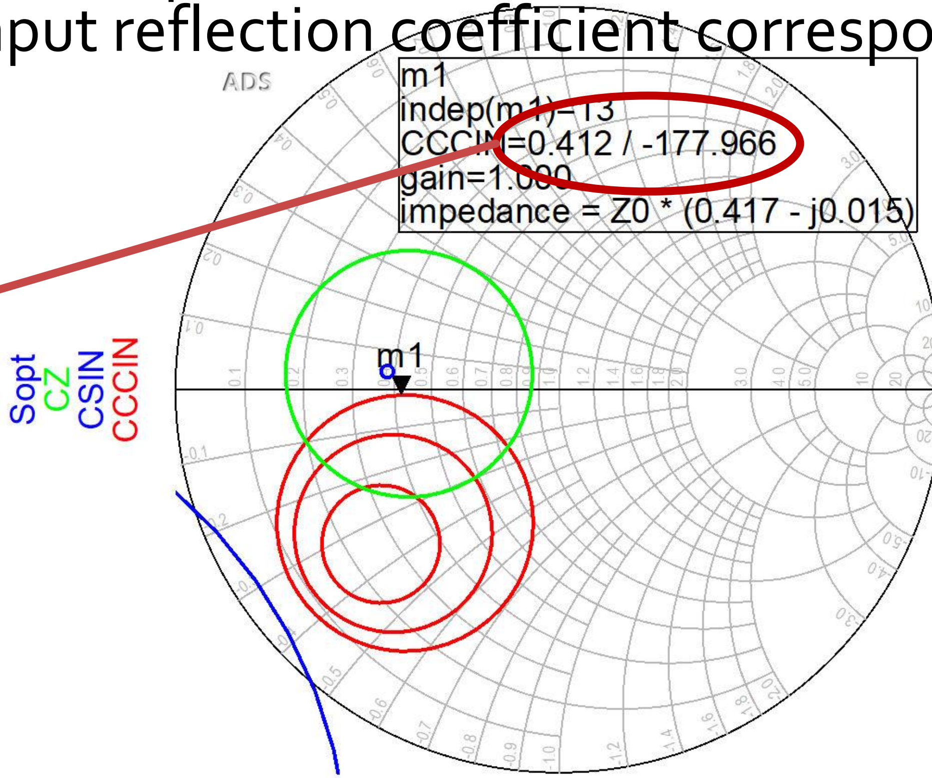
- Connecting the amplifier (transistor) directly to the source with Zo generate a reflection coefficient seen towards the source equal with **0** (complex number, $\Gamma_0 = 0 + 0 \cdot j$)
  - most of the time this reflection coefficient does not offer optimum noise/gain



Matching – 2

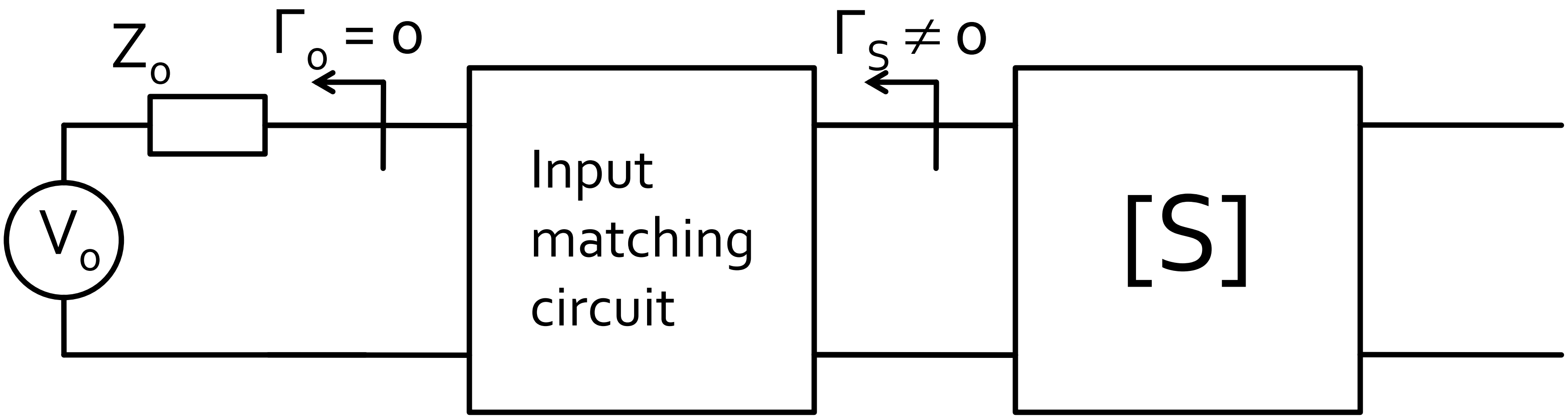
- We plot on the complex plane (Smith Chart) the stability/gain/noise circles (depending on the particular application)
- We choose a point with a suitable position relative to these circles (also application dependent)
- We determine the input reflection coefficient corresponding to this point, Γ_S

$$\Gamma_S = 0.412\angle -177.966°$$



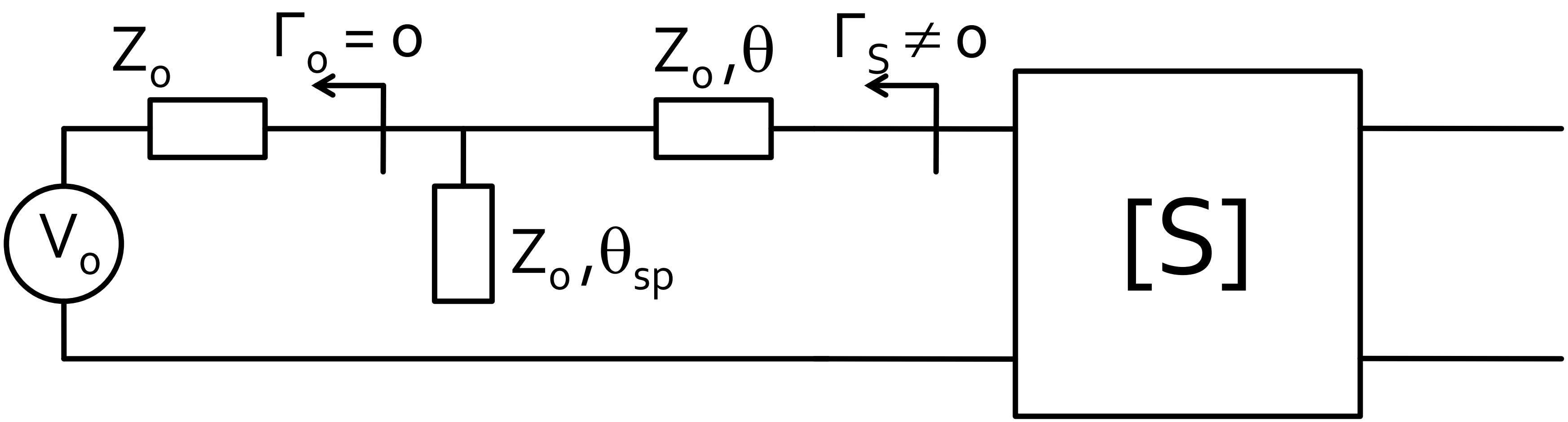
Matching – 3

- We insert the input matching circuits which allows the transistor to see towards the source the previously determined reflection coefficient Γ_S



Matching – 4

- Easiest to design matching section consists in the insertion of (in order from the transistor towards the Z_o source):
  - a series Z_o line, with electrical length θ
  - a shunt stub, open-circuited, made from a Z_o line, with electrical length θ_{sp}



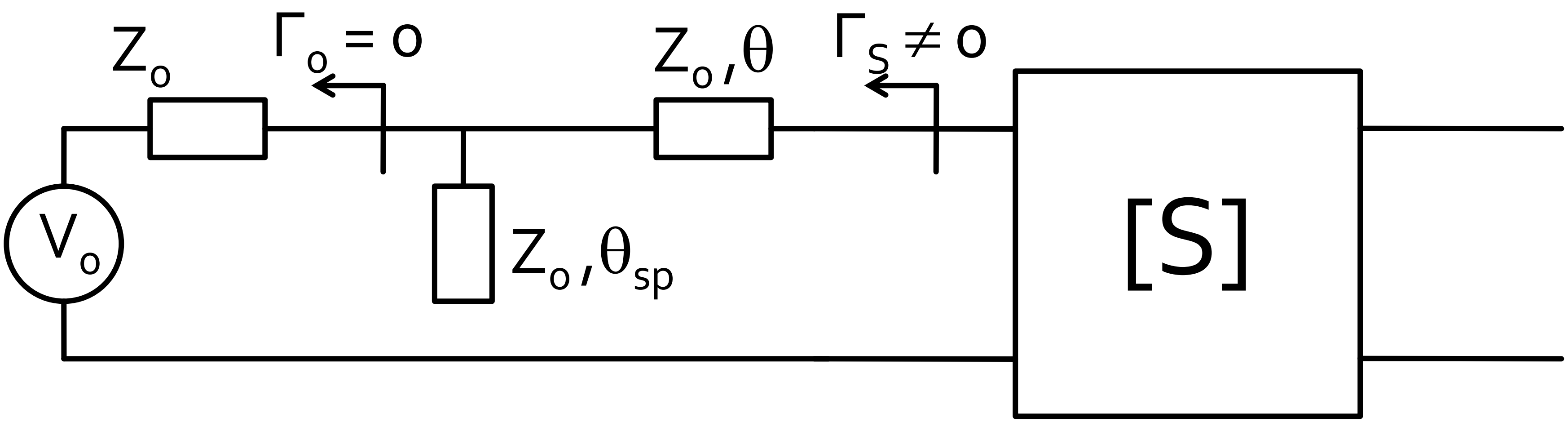
Matching – 5

- Computation depends solely on Γ_S (magnitude and phase)

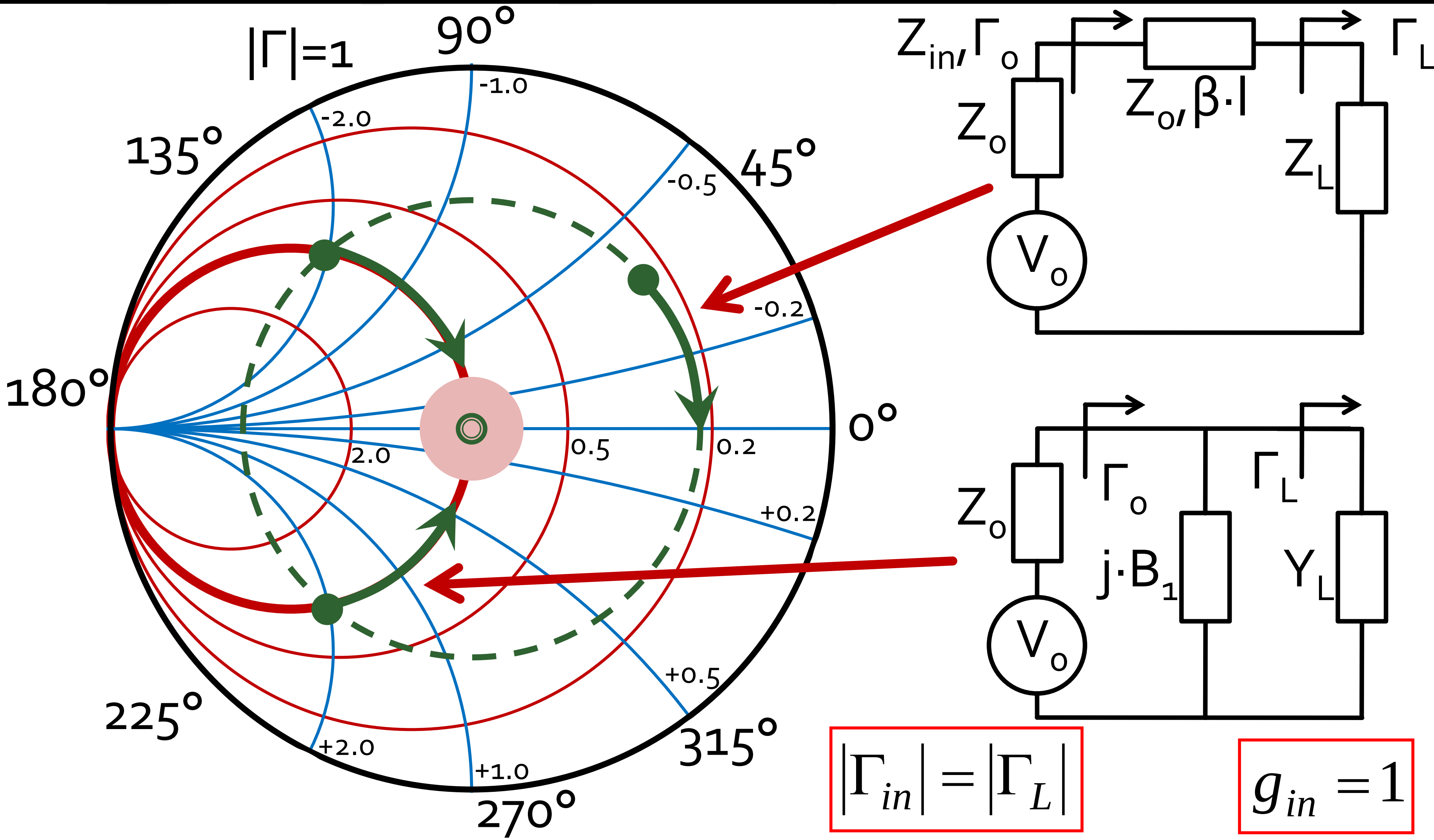
$$\cos(\varphi_S + 2\theta) = -|\Gamma_S|$$

$$\tan\theta_{sp} = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}$$

- The sign (+/-) chosen for the series line equation imposes the sign used for the shunt stub equation



Shunt stub matching, L7



$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$g_{in} = 1$$

Example, LNA @ 5 GHz

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - $S_{11} = 0.64\angle 139^\circ$
  - $S_{12} = 0.119\angle -21^\circ$
  - $S_{21} = 3.165\angle 16^\circ$
  - $S_{22} = 0.22\angle 146^\circ$
  - $F_{min} = 0.54$ (**tipic [dB]**)
  - $\Gamma_{opt} = 0.45\angle 174^\circ$
  - $r_n = 0.03$

```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt    GAMMA OPT      RN/Zo
!GHZ     dB     MAG     ANG      -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

Example, LNA @ 5 GHz

- Low Noise Amplifier
- At the input matching a compromise is required between:
  - noise (~~input~~ constant noise figure circles)
  - gain (input constant gain circles)
  - stability (input stability circle)
- At the output matching noise **is not influenced**. A compromise is required between :
  - gain (output constant gain circles)
  - stability (output stability circle)

Example, LNA @ 5 GHz

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1-|S_{11}|^2\right) \cdot \left(1-|S_{22}|^2\right)} = 0.094$$

$$-0.783\,dB < G_T[dB] - G_{TU}[dB] < 0.861\,dB$$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2} = 17.83$$

$$G_{TU\max}[dB] = 12.511\,dB$$

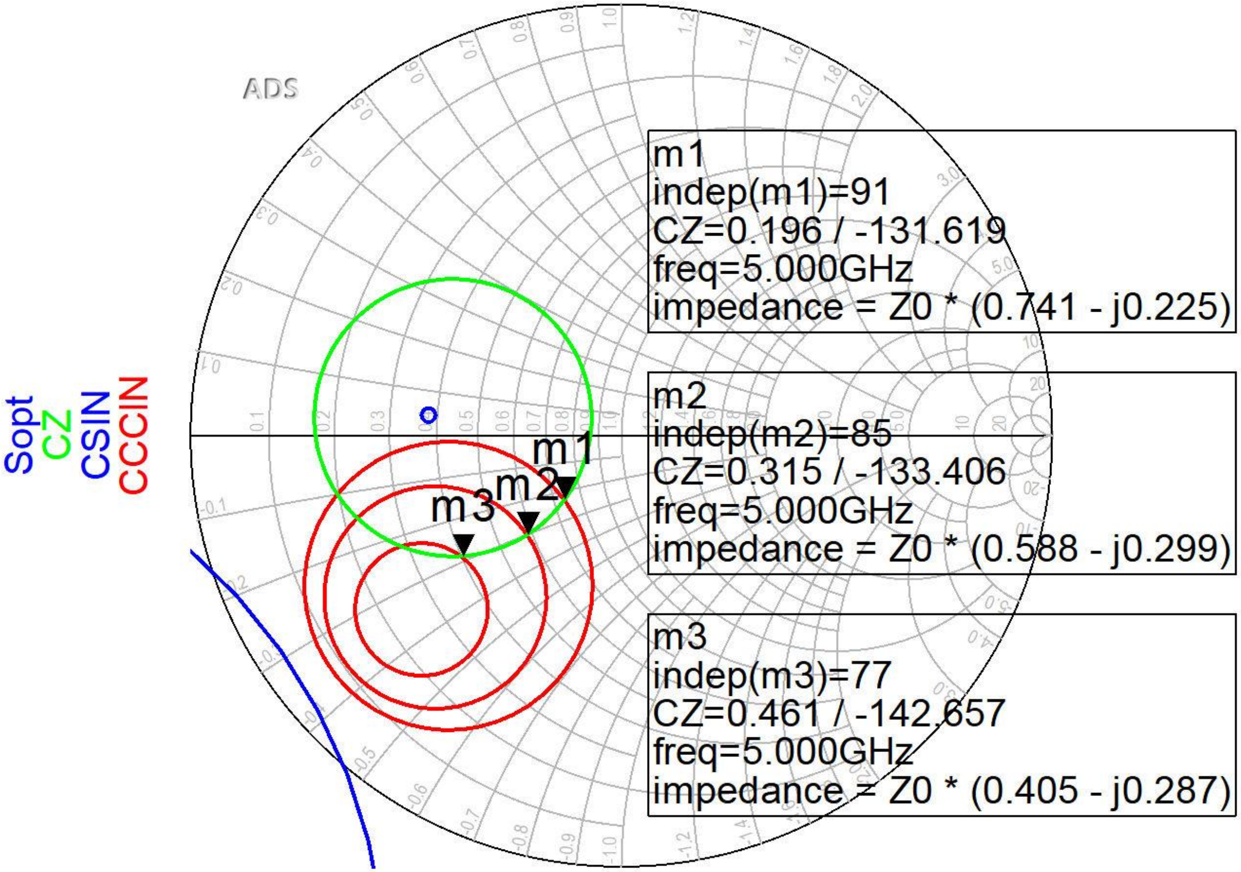
$$G_0 = |S_{21}|^2 = 10.017 = 10.007\,dB$$

$$G_{S\max} = \frac{1}{1-|S_{11}|^2} = 1.694 = 2.289\,dB$$

$$G_{L\max} = \frac{1}{1-|S_{22}|^2} = 1.051 = 0.215\,dB$$

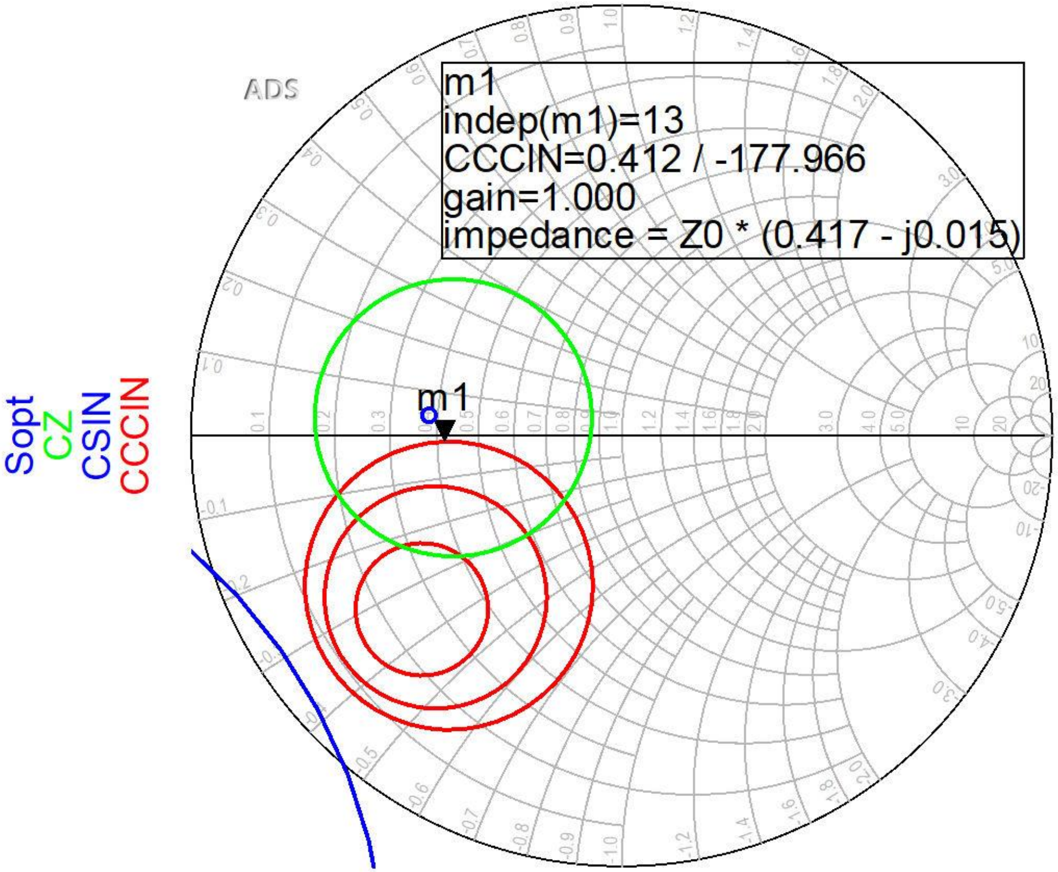
- In this particular case G_{Lmax} = 0.21 dB, the transistor could be used directly connected to the 50Ω load
- The absence of the output matching circuit **is not** recommended. While the attainable power gain is low, it's absence eliminates the possibility to use it to compensate an improper gain generated by the noise optimization of the input matching circuit

Input matching circuit



- For the input matching circuit
  - noise circle CZ: 0.75dB
  - input constant gain circles CCCIN: 1dB, 1.5dB, 2 dB
- We choose (small Q → wide bandwidth) position m1

Input matching circuit



- If we can afford a 1.2dB decrease of the input gain for better NF,Q (Gs = 1 dB), position m1 above is better
- We obtain better (smaller) NF

Input matching circuit

- Position m1 in complex plane (Smith Chart)

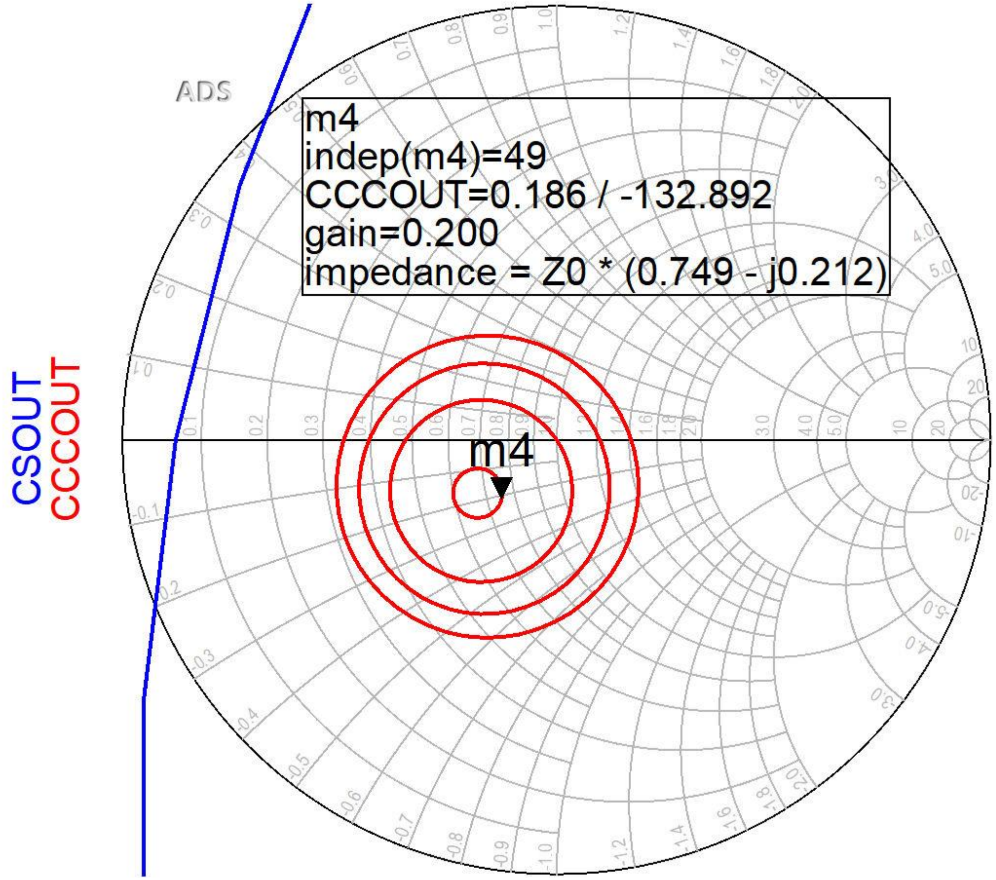
$$\Gamma_S = 0.412\angle -178^\circ \qquad |\Gamma_S| = 0.412; \quad \varphi = -178^\circ$$

$$\cos(\varphi + 2\theta) = -|\Gamma_S| \qquad \mathrm{Im}[y_S(\theta)] = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\cos(\varphi + 2\theta) = -0.412 \Rightarrow \quad (\varphi + 2\theta) = \pm 114.33^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +114.33^\circ \\ -114.33^\circ \end{cases} \quad \theta = \begin{cases} 146.2^\circ \\ 31.8^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -0.904 \\ +0.904 \end{cases} \quad \theta_{sp} = \begin{cases} 137.9^\circ \\ 42.1^\circ \end{cases}$$

Output matching circuit



- output constant gain circles CCCOUT: -0.4dB, -0.2dB, 0dB, +0.2dB
- the lack of noise restrictions allows optimization for better gain (close to maximum – position m4)

Output matching circuit

- Position m4 in complex plane (Smith Chart)

$$\Gamma_L = 0.186\angle -132.9^\circ \qquad |\Gamma_L| = 0.186; \quad \varphi = -132.9^\circ$$

$$\cos(\varphi + 2\theta) = -|\Gamma_L| \qquad \mathrm{Im}[y_L(\theta)] = \frac{-2\cdot|\Gamma_L|}{\sqrt{1-|\Gamma_L|^2}} = -0.379$$

$$\cos(\varphi + 2\theta) = -0.186 \Rightarrow \quad (\varphi + 2\theta) = \pm 100.72^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +100.72^\circ \\ -100.72^\circ \end{cases} \quad \theta = \begin{cases} 116.8^\circ \\ 16.1^\circ \end{cases} \quad \mathrm{Im}[y_L(\theta)] = \begin{cases} -0.379 \\ +0.379 \end{cases} \quad \theta_{sp} = \begin{cases} 159.3^\circ \\ 20.7^\circ \end{cases}$$

LNA

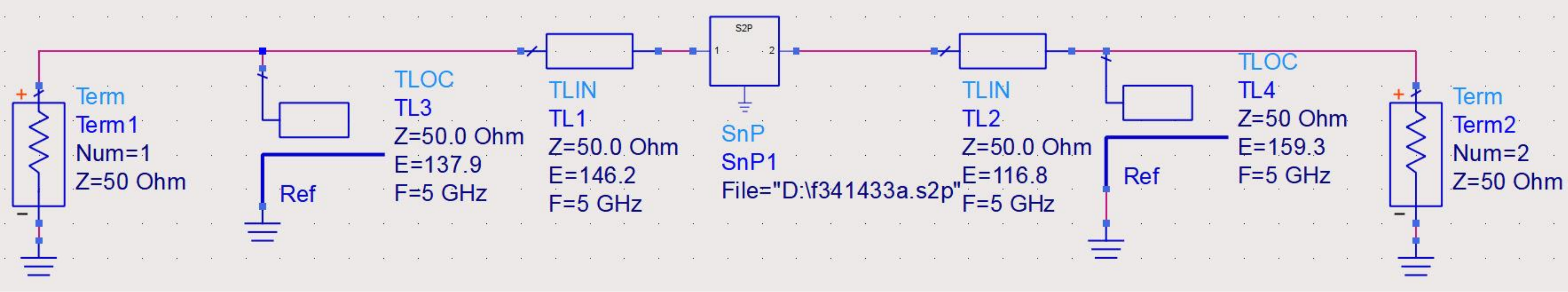
- We estimate a gain (in unilateral assumption, ±0.9 dB)

$$G_T[dB] = G_S[dB] + G_0[dB] + G_L[dB]$$

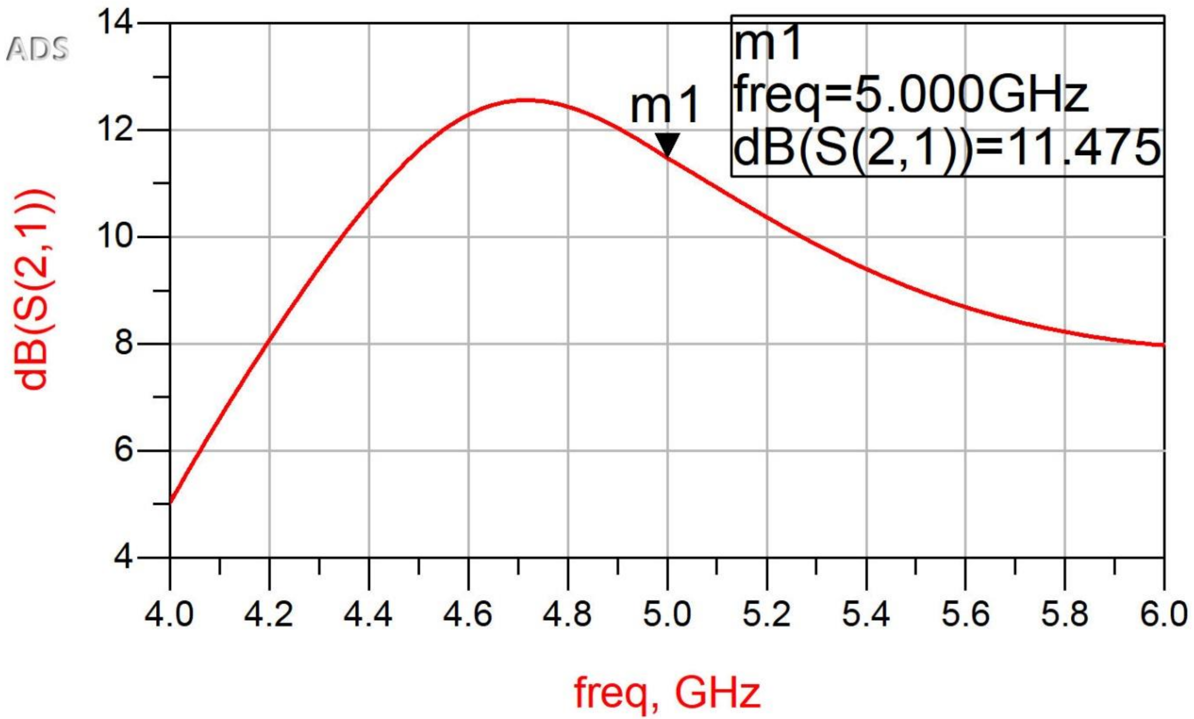
$$G_T[dB] = 1\ dB + 10\ dB + 0.2\ dB = 11.2\ dB$$

- We estimate a noise factor well bellow 0.75dB (quite close to the minimum ~0.6 dB)

ADS

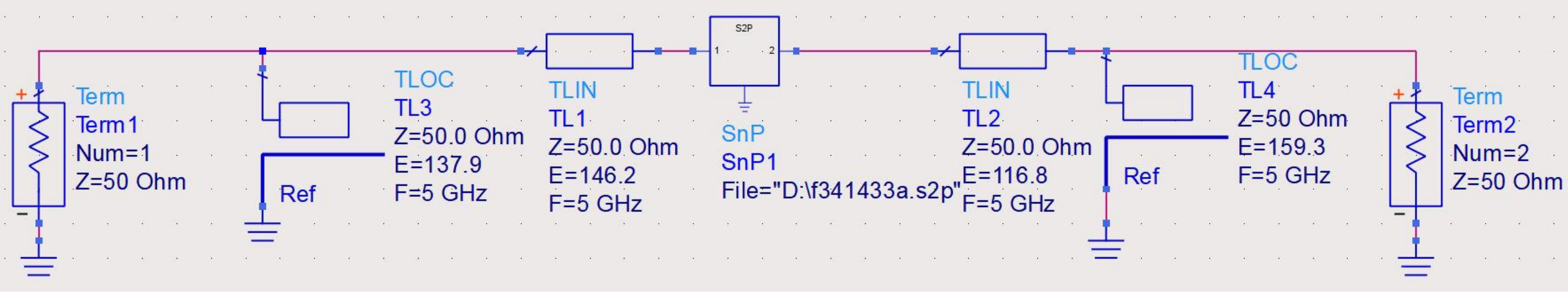


Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=137.9
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=146.2
F=5 GHz
S2P
SnP
SnP1
File="D:\f341433a.s2p"
TLIN
TL2
Z=50.0 Ohm
E=116.8
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=159.3
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm

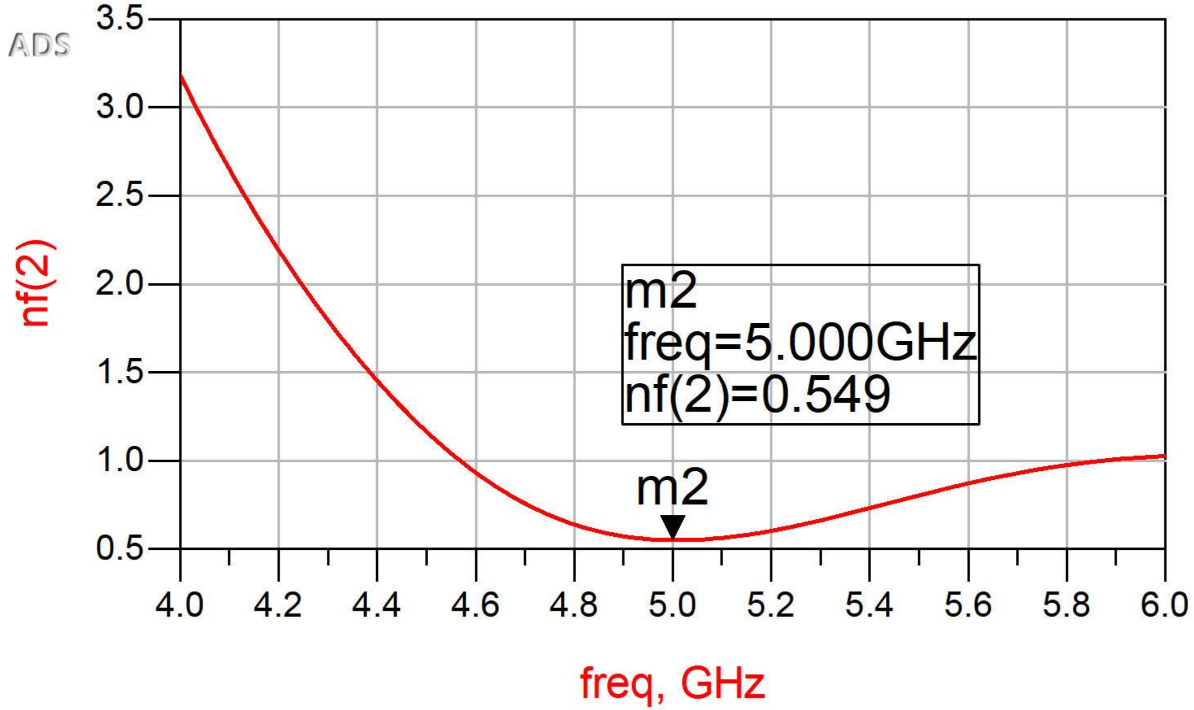


ADS
m1
freq=5.000GHz
dB(S(2,1))=11.475
m1
dB(S(2,1))
14
12
10
8
6
4
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz

ADS



Term
Term1
Num=1
Z=50 Ohm
TLOC
TL3
Z=50.0 Ohm
E=137.9
F=5 GHz
Ref
TLIN
TL1
Z=50.0 Ohm
E=146.2
F=5 GHz
S2P
SnP
SnP1
File="D:\f341433a.s2p"
TLIN
TL2
Z=50.0 Ohm
E=116.8
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=159.3
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm

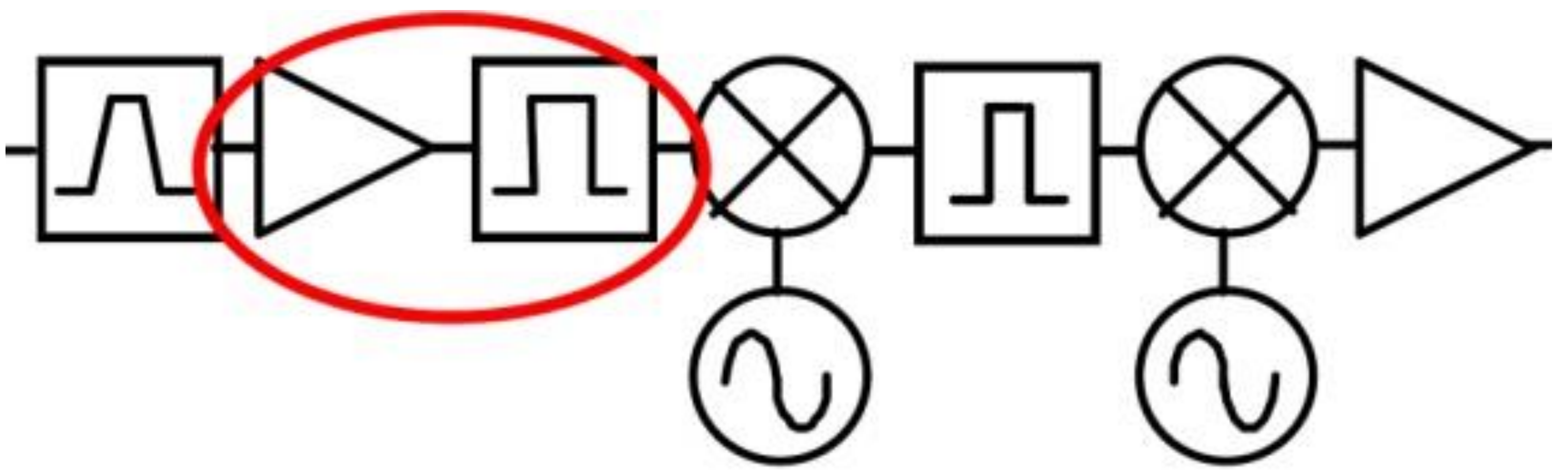


ADS
nf(2)
m2
freq=5.000GHz
nf(2)=0.549
m2
freq, GHz

Microwave Filters

Assignment

- this structure is frequently encountered in radiocommunication systems



$$P_n = kTB$$

Microwave Filters

- Two ways of implementing filters in microwave frequency range
  - microwave specific structures (coupled lines, dielectric resonators, periodic structures)
  - **filter synthesis** with lumped elements followed by implementation with transmission lines
- the first strategy leads to more efficient filters but:
  - has lower generality
  - design is often difficult (lack of analytical relationships)

Filter synthesis

- Filter is designed with lumped elements (L/C) followed by implementation with distributed elements (transmission lines)
  - general
  - analytical relationships easy to implement on the computer
  - efficient
- The preferred procedure is **insertion loss method**

Insertion loss method

$$P_{LR} = \frac{P_S}{P_L} = \frac{1}{1-\left|\Gamma(\omega)\right|^2}$$

- $|\Gamma(\omega)|^2$ is an even function of ω

$$\left|\Gamma(\omega)\right|^2 = \frac{M\left(\omega^2\right)}{M\left(\omega^2\right)+N\left(\omega^2\right)}$$

$$P_{LR} = 1+\frac{M\left(\omega^2\right)}{N\left(\omega^2\right)}$$

- Choosing M and N polynomials appropriately leads to a filter with a completely specified frequency response

Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
  - in the passband (pass all frequencies)
  - in the stopband (reject all frequencies)

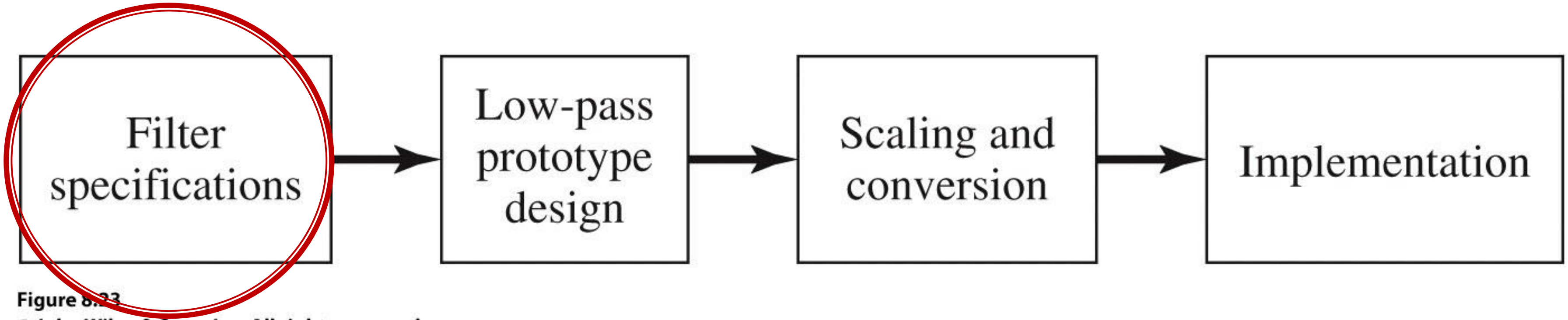
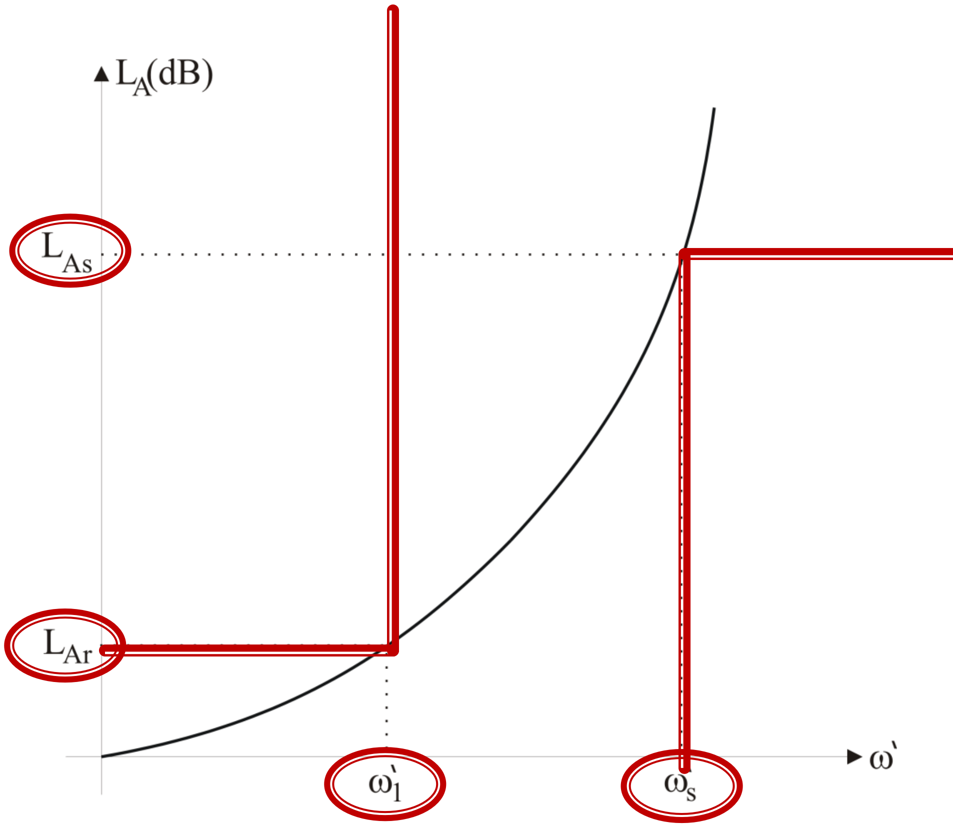


Figure 8.23

Filter specifications

- Attenuation
  - in passband
  - in stopband
  - most often in **dB**
- Frequency range
  - passband
  - stopband
  - cutoff frequency ω_1' usually normalized (**= 1**)



Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
  - low-pass, high-pass, bandpass, or bandstop

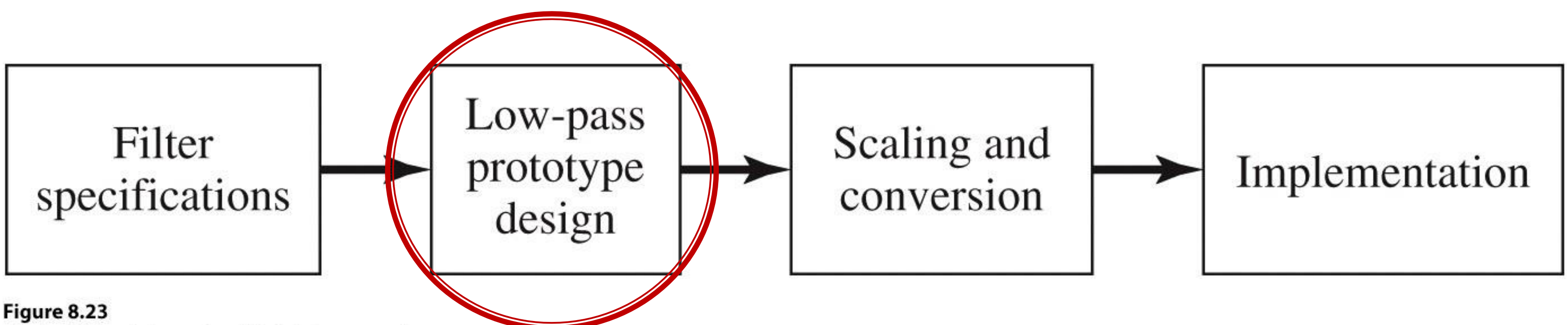


Figure 8.23

Practical low-pass prototypes responses

- **Maximally flat filters** (Butterworth, binomial): provide the flattest possible passband response
- **Equal ripple filters** (Chebyshev): provide a sharper cutoff but the passband response will have ripples
- **Elliptic function filters**, they have equal-ripple responses in the passband as well as in the stopband,
- **Linear phase filters**, offer linear phase response in the passband to avoid signal distortion (important in some applications)

Maximally Flat/Equal ripple LPF Prototype

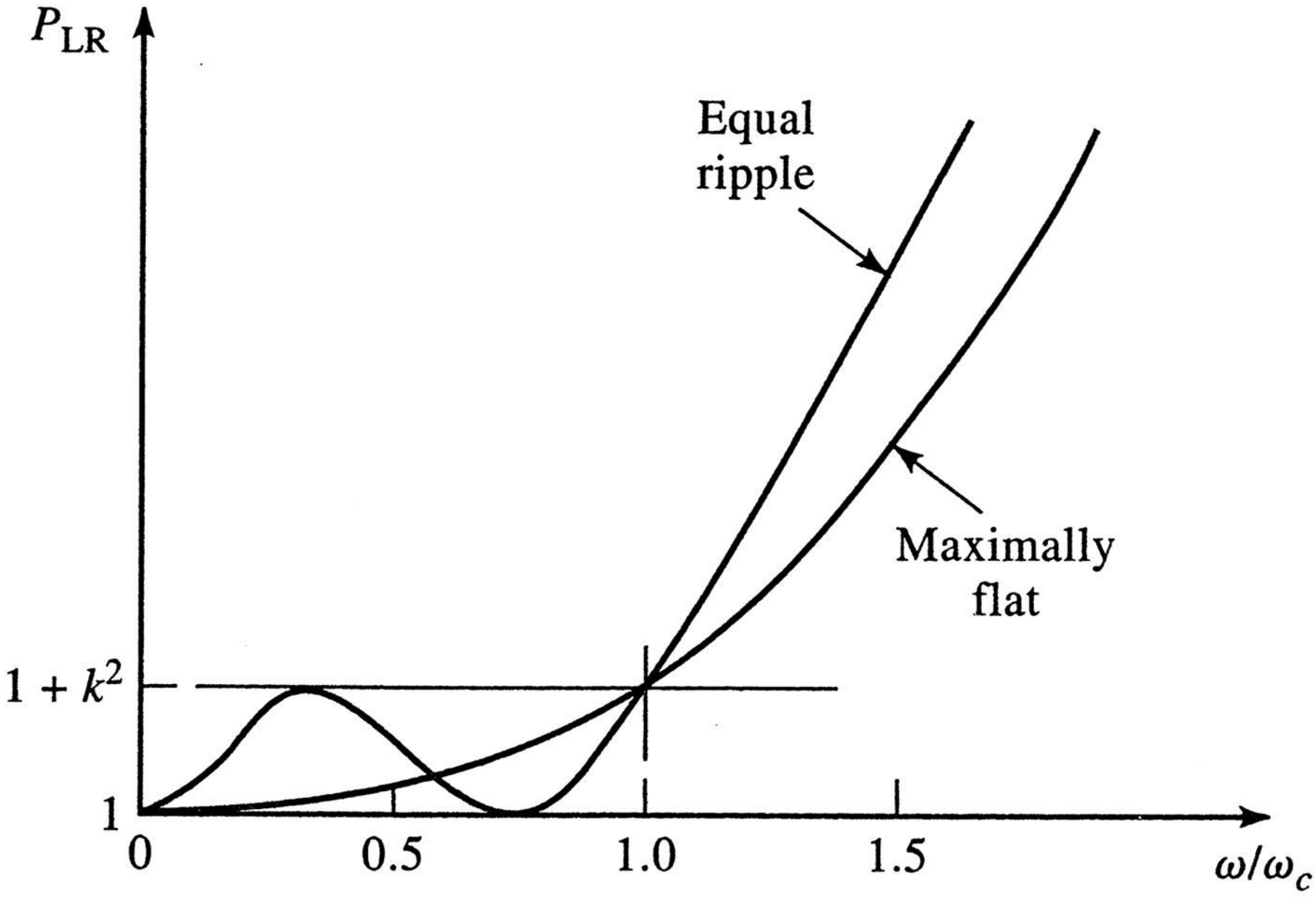


Figure 8.21

Elliptic function LPF Prototype

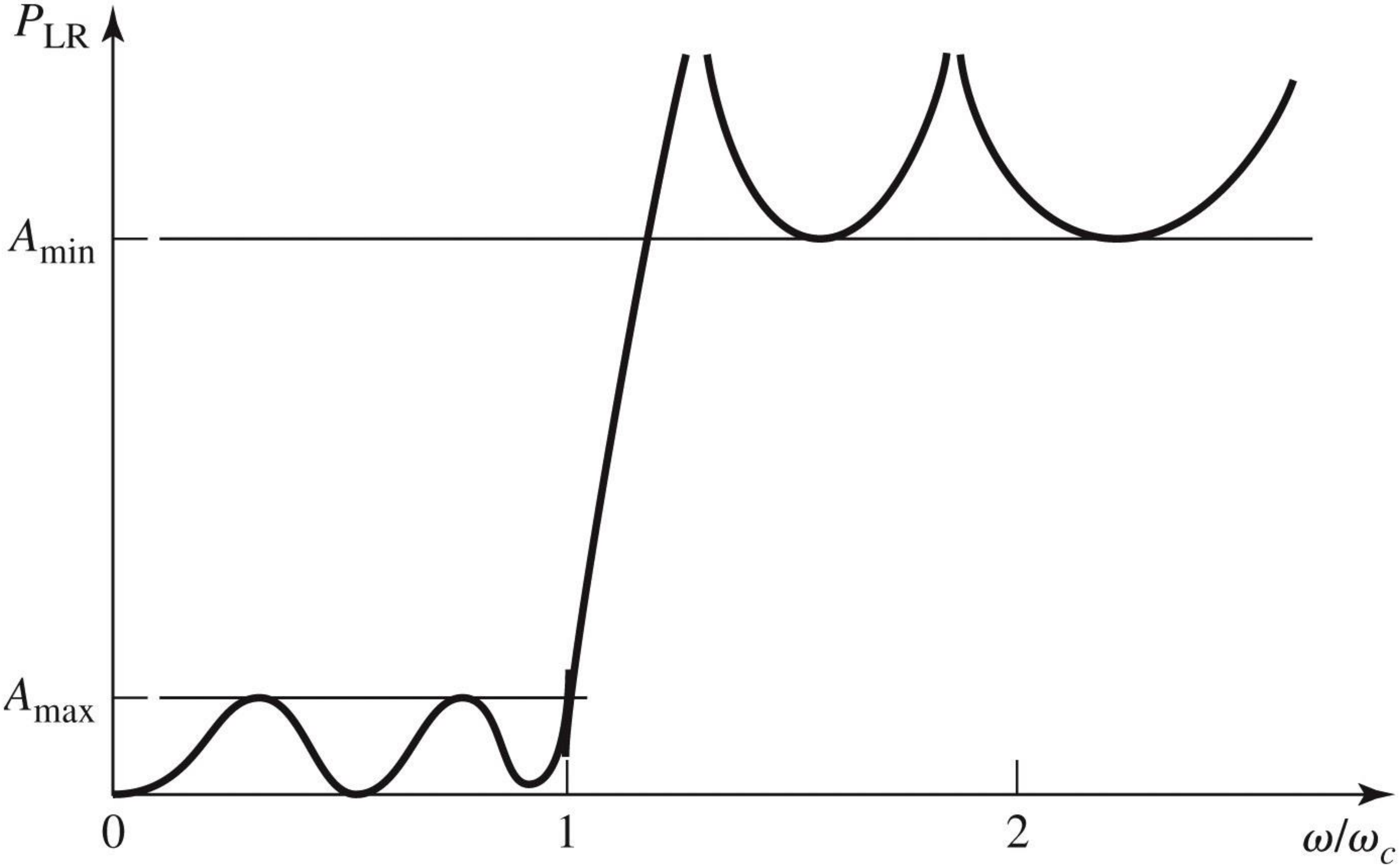


Figure 8.22

Maximally Flat LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot \left(\frac{\omega}{\omega_c}\right)^{2N}$$

- For $\omega >> \omega_c$

$$P_{LR} \approx k^2 \cdot (\omega/\omega_c)^{2N}$$

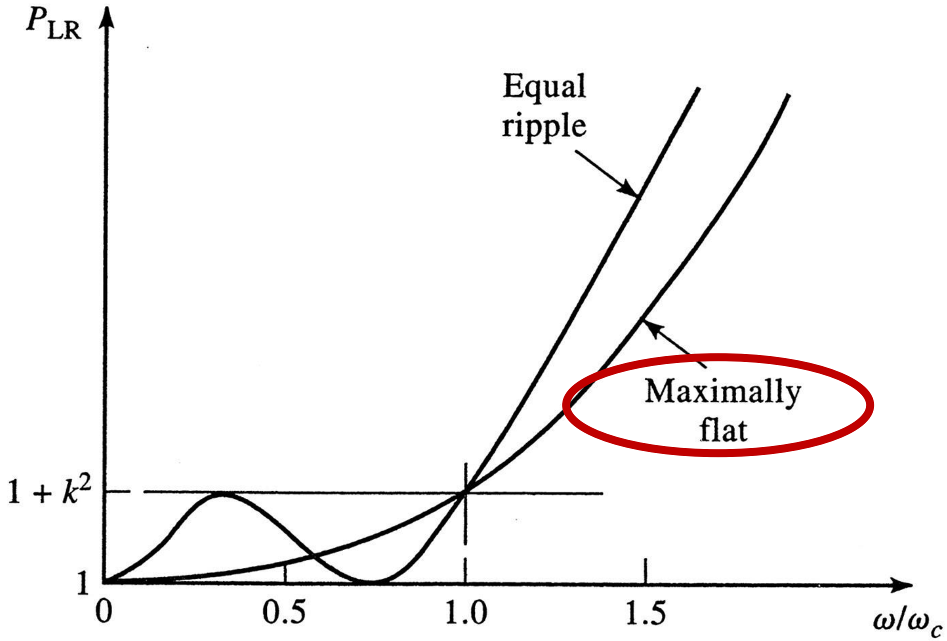


Figure 8.21

- attenuation increases at a rate of 20·N dB/decade
- k gives the attenuation at cutoff frequency (3dB cutoff imposes k = 1)

Equal Ripple LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot T_N^2\left(\frac{\omega}{\omega_c}\right)$$

- For $\omega >> \omega_c$

$$P_{LR} \approx \frac{k^2}{4} \cdot \left(\frac{2 \cdot \omega}{\omega_c}\right)^{2N}$$

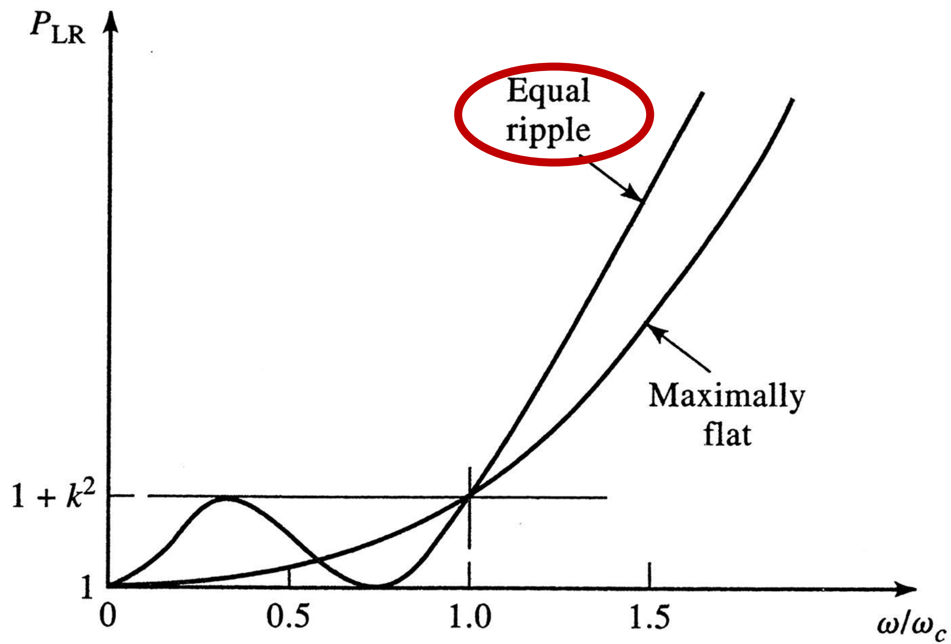


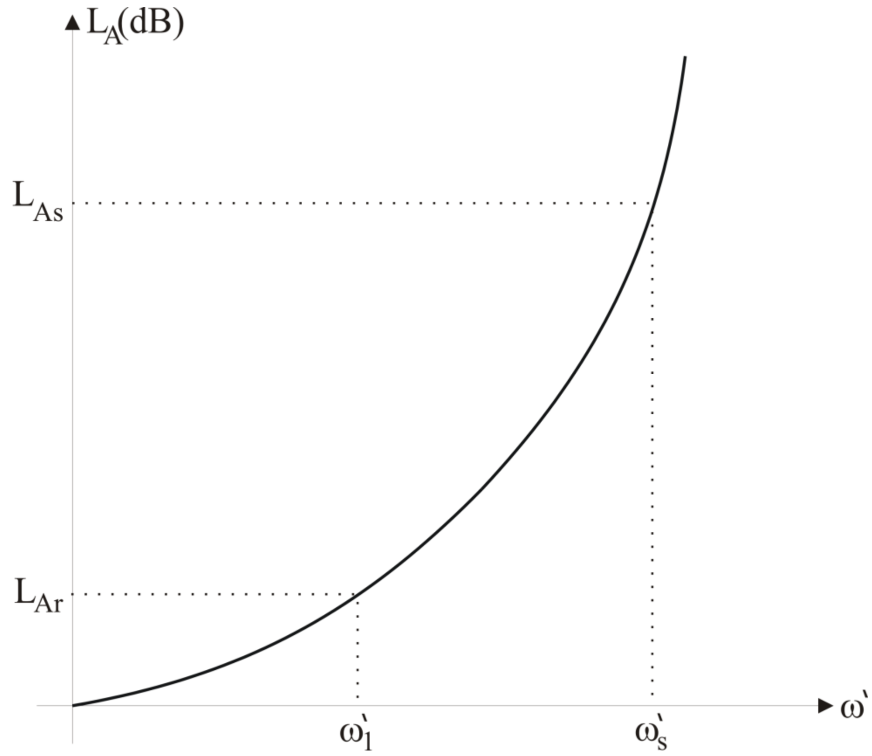
Figure 8.21

- attenuation increases at a rate of 20·N dB/decade (**also**)
- attenuation is $(2^{2N})/4$ greater than the binomial response at any given frequency where $\omega >> \omega_c$
- the passband ripples: $1 + k^2$, k gives the ripple

Order (N) of the Maximally Flat filter

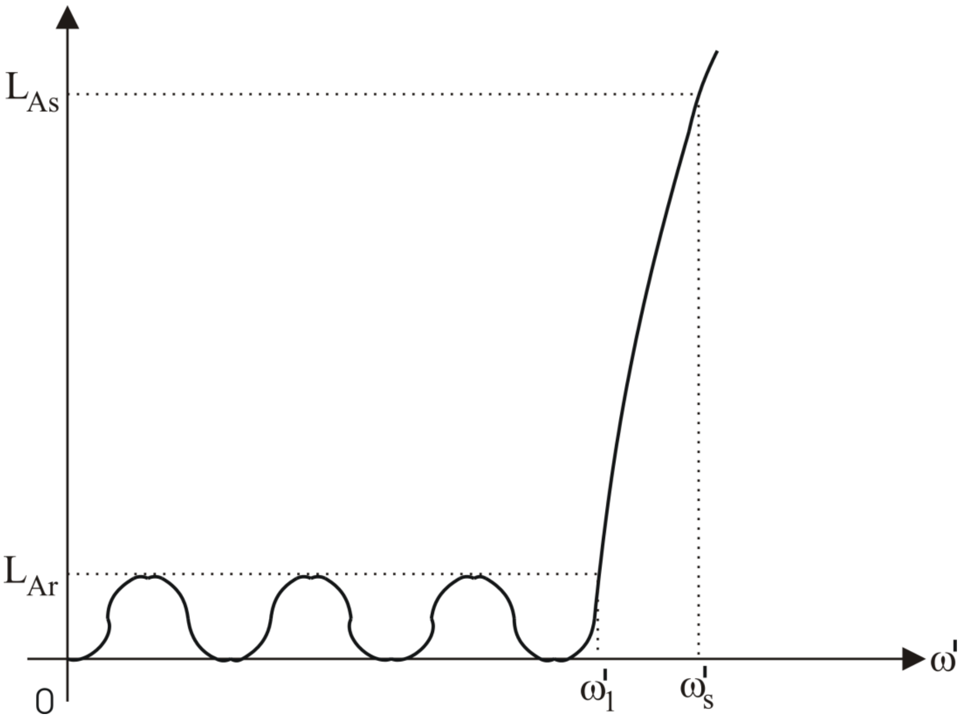
$$n \geq \frac{\log\left(\dfrac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}\right)}{2\cdot\log\dfrac{\omega'_s}{\omega'_1}}$$

- **!**attenuations in **dB**



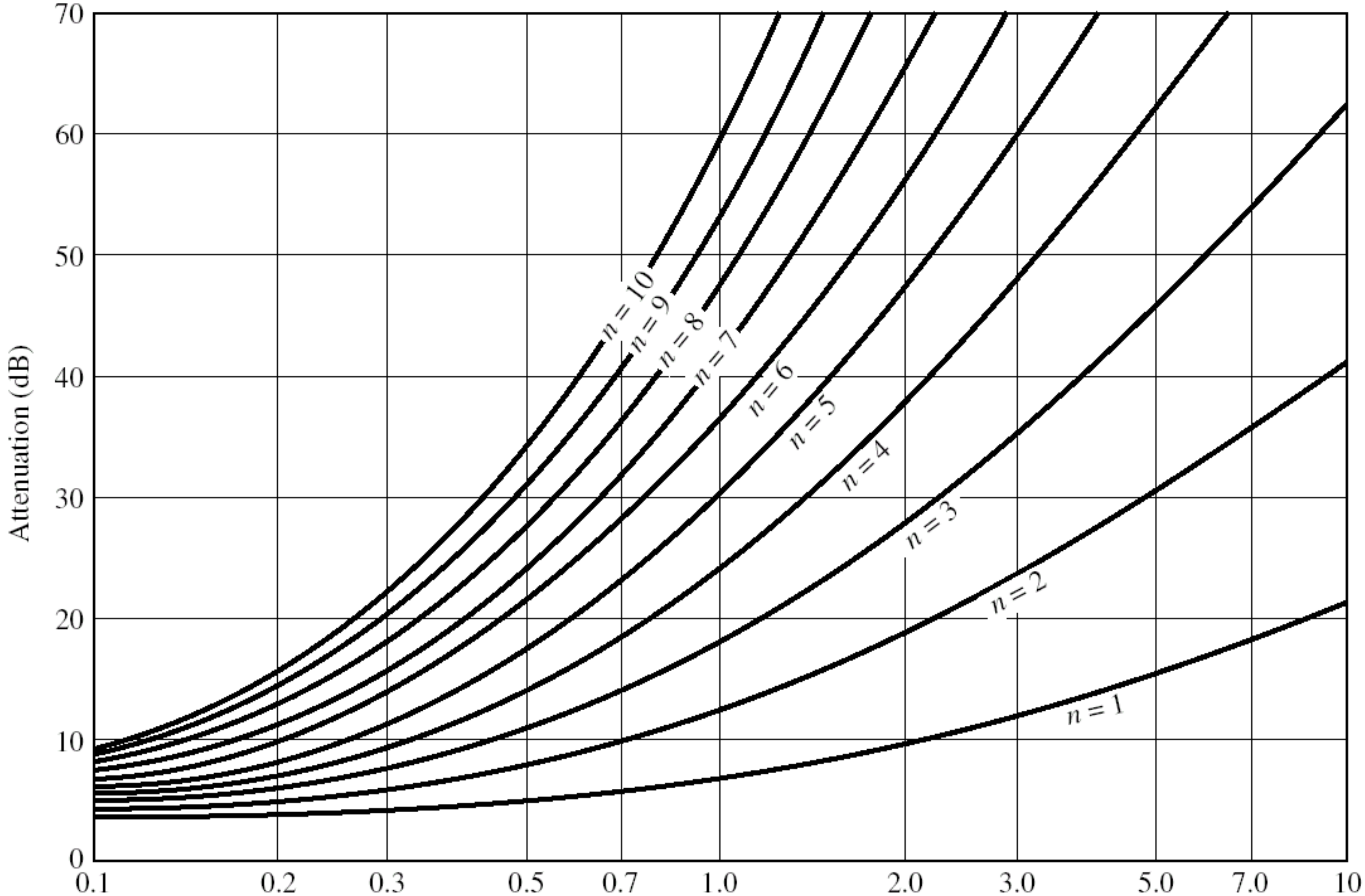
Order (N) of the Equal Ripple filter

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\frac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}}\right)}{\cosh^{-1}\left(\frac{\omega'_s}{\omega'_1}\right)}$$



- !attenuations in **dB**

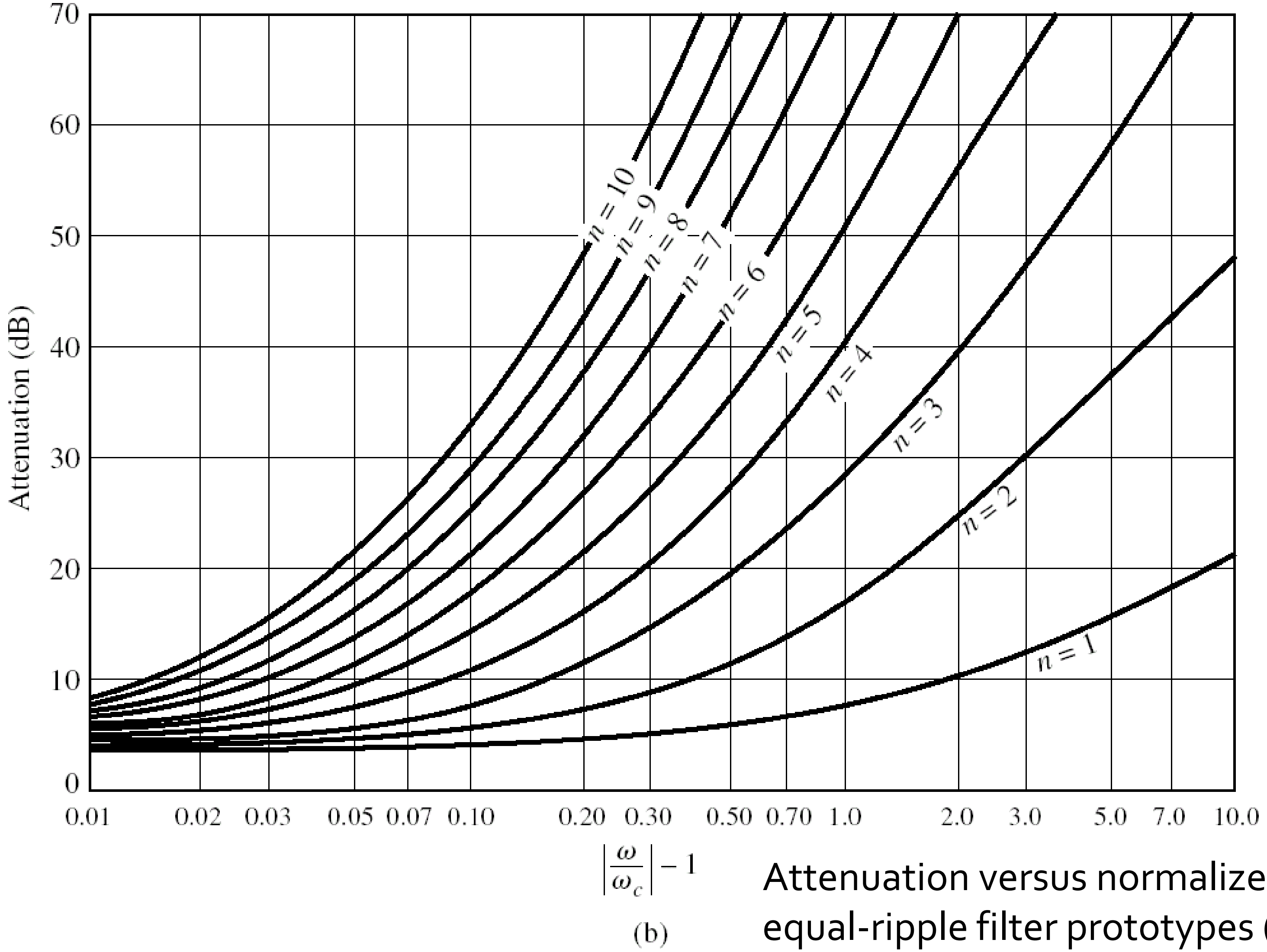
Maximally flat filter prototypes



$\left|\frac{\omega}{\omega_c}\right| - 1$

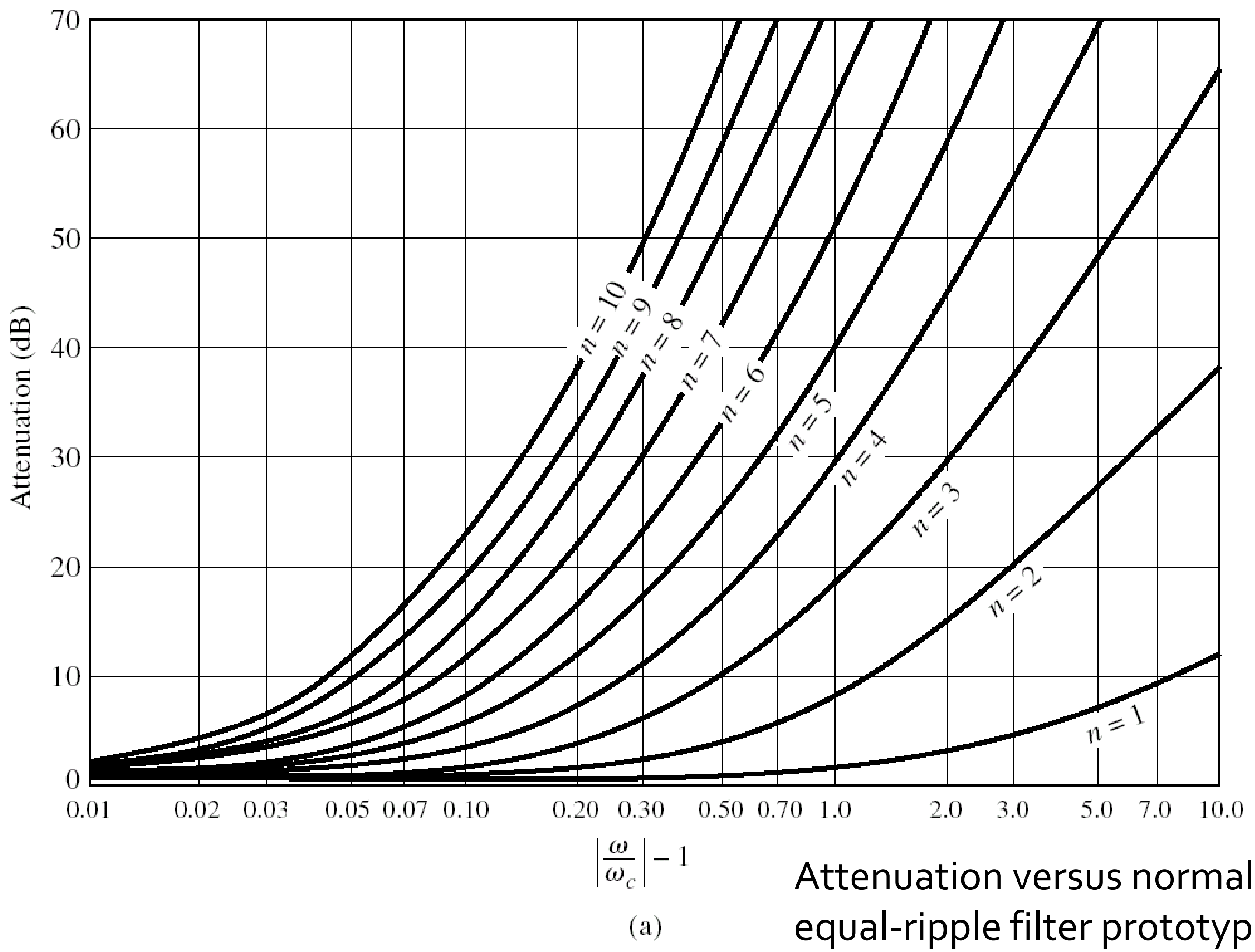
Attenuation versus normalized frequency for maximally flat filter prototypes

3 dB Equal-ripple filter prototypes



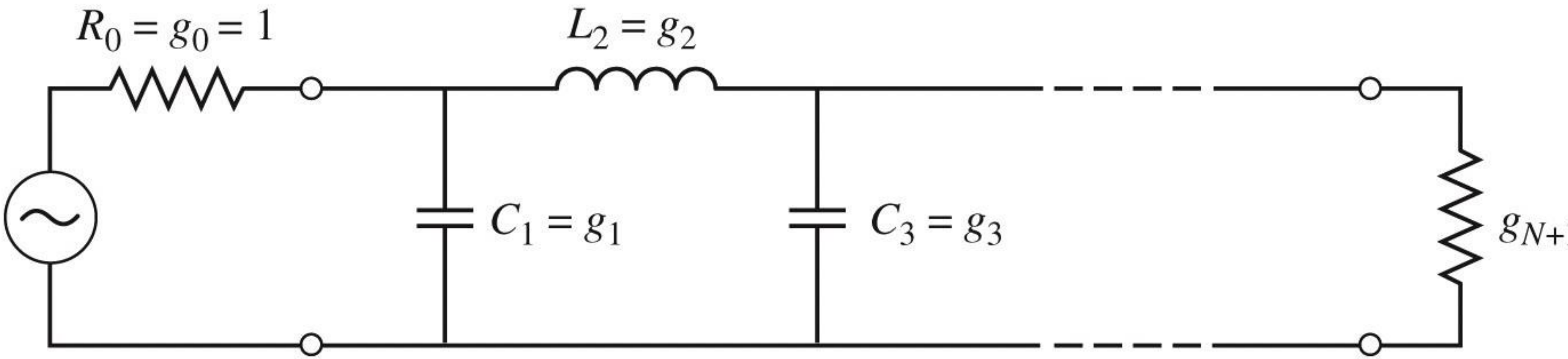
Attenuation versus normalized frequency for equal-ripple filter prototypes (3dB)

0.5 dB Equal-ripple filter prototypes

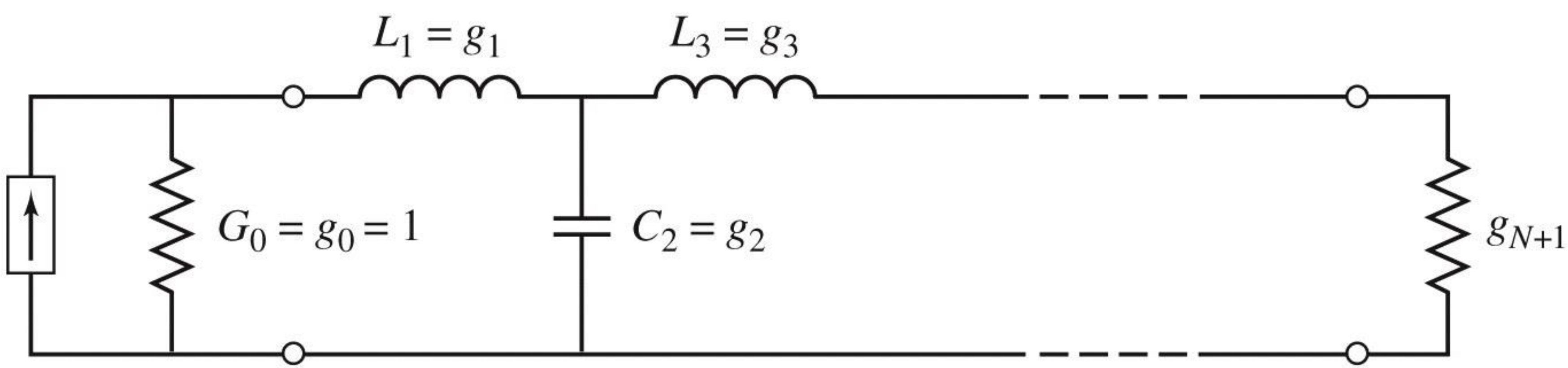


Attenuation versus normalized frequency for equal-ripple filter prototypes (0.5dB)

Prototype Filters



(a)



(b)

Prototype Filters

- Prototype filters are:
  - Low-Pass Filters (**LPF**)
  - cutoff frequency **ω_o = 1 rad/s** (f_o = 0.159 Hz)
  - connected to a source with **R = 1Ω**
- The number of reactive elements (L/C) is the order of the filter (N)
- Reactive elements are alternated: series L / shunt C
- There two prototypes with the same response, a prototype beginning with a shunt C element, and a prototype beginning with a series L element

Prototype Filters

- We define filter parameters g_i, i=0,N+1
- g_i are the element values in the prototype filter

$$g_0 = \begin{cases} \text{generator resistance } R'_0 \ \textit{if} \ \ g_1 = C'_1 \\ \text{generator conductance } G'_0 \ \textit{if} \ \ g_1 = L'_1 \end{cases}$$

$$g_k\Big|_{k=\overline{1,N}} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance } R'_{N+1} \ \textit{if} \ \ g_N = C'_N \\ \text{load conductance } G'_{N+1} \ \textit{if} \ \ g_N = L'_N \end{cases}$$

Maximally Flat LPF Prototype

- Formulas for filter parameters

$$g_0 = 1$$

$$g_k = 2 \cdot \sin\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N}\right] \quad , \quad k = 1, N$$

$$g_{N+1} = 1$$

Maximally Flat LPF Prototype

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Table 8.3

Equal-ripple LPF Prototype

- Formulas for filter parameters (iterative)

$$a_k = \sin\left[\frac{(2\cdot k - 1)\cdot \pi}{2\cdot N}\right] \quad , \quad k = 1, N \qquad\qquad \beta = \ln\left(\coth\frac{L_{Ar}}{17.37}\right)$$

$$\gamma = \sinh\left(\frac{\beta}{2\cdot N}\right) \qquad\qquad b_k = \gamma^2 + \sin^2\left(\frac{k\cdot\pi}{N}\right) \quad , \quad k = 1, N$$

$$g_1 = \frac{2\cdot a_1}{\gamma}$$

$$g_k = \frac{4\cdot a_{k-1}\cdot a_k}{b_{k-1}\cdot g_{k-1}} \quad , \quad k = 2, N$$

$$g_{N+1} = \begin{cases} 1 & \text{for odd } N \\ \coth^2\left(\frac{\beta}{4}\right) & \text{for even } N \end{cases}$$

TABLE 8.4 **Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)**

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass.,1980, with permission.

Table 8.4

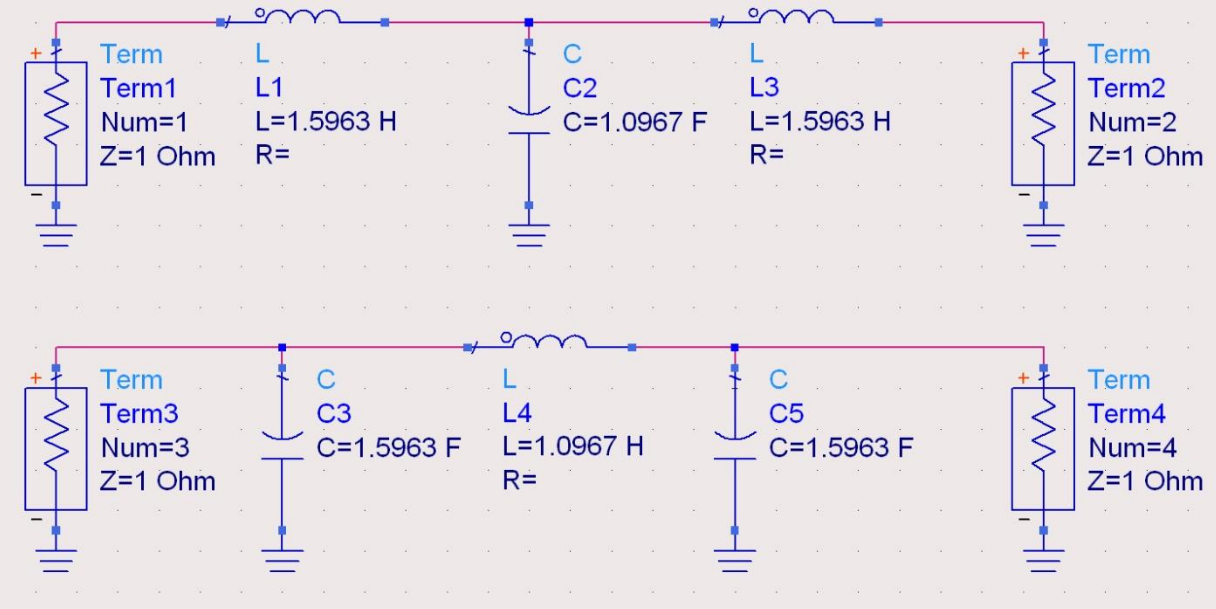
- For even N order of the filter (N = 2, 4, 6, 8 …) equal-ripple filters **must** closed by a load impedance **$g_{N+1} \neq 1$**
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial …) to **$g_L = 1$**

Example

- Design a **3rd order** ~~bandpass~~ filter with **0.5 dB ripples** in passband. ~~The center frequency of the filter should be 1 GHz. The fractional bandwidth of the passband should be 10%, and the impedance 50Ω~~.

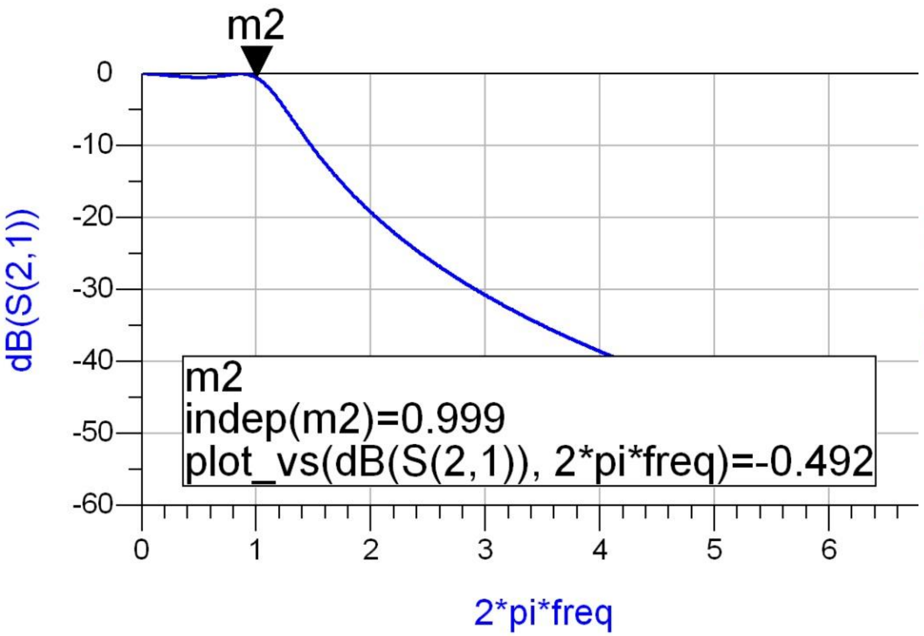
LPF Prototype

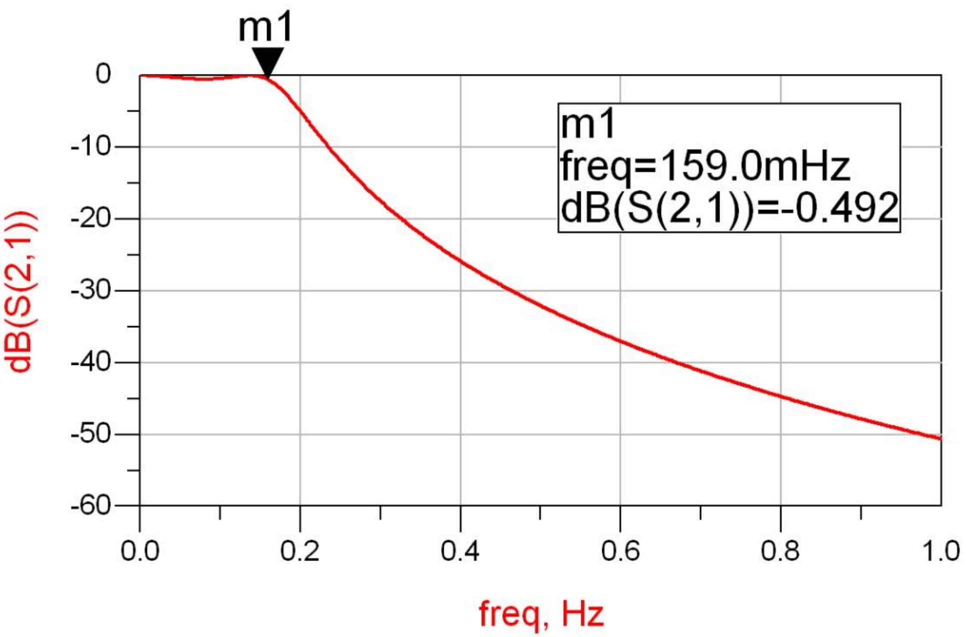
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 = 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Impedance and Frequency Scaling

- After computing prototype filter's elements:
  - Low-Pass Filters (**LPF**)
  - cutoff frequency **ω_o = 1 rad/s** (f_o = 0.159 Hz)
  - connected to a source with **R = 1Ω**
- component values can be scaled in terms of impedance and frequency

Impedance and Frequency Scaling

- LPF Prototype is only used as an intermediate step
  - Low-Pass Filter (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω

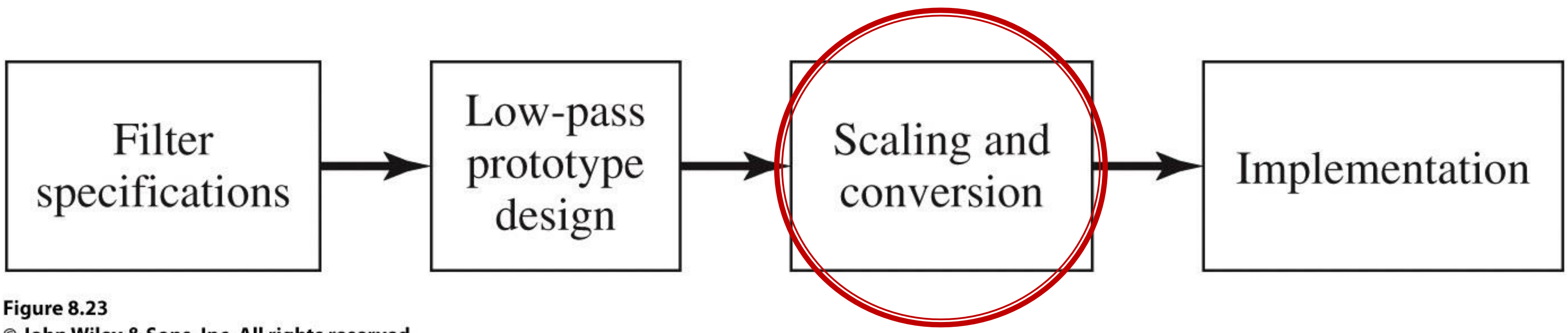


Figure 8.23

Impedance Scaling

- To design a filter which will work with a source resistance of R_o we multiplying all the impedances of the prototype design by R_o ("**'**" denotes scaled values)

$$R_s' = R_0 \cdot (R_s = 1)$$

$$R_L' = R_0 \cdot R_L$$

$$L' = R_0 \cdot L$$

$$C' = \frac{C}{R_0}$$

Contact

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